Quantum
Mechanical &
Electromagnetic
Systems
Modelling Lab

Reduced-Order Stochastic Testing of Interconnects Subject to Line Edge Roughness

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quest.

Outline

Motivation

Methodology

1. Finite-element based eigenvalue solver
2. Sparse grid stochastic testing
3. Reduced-order stochastic testing

Examples

Conclusions

Motivation

Motivation

Line edge roughness

Higher operating frequencies & increased heterogenous integration causes stronger influence on physical phenomena

Line edge roughness (LER) is irregular profile of conductors owing to production process

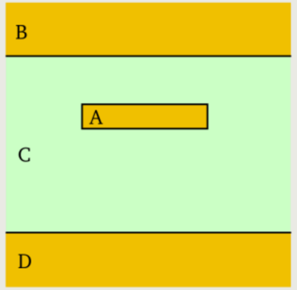
Impact becomes stronger due to more developed skin effect

Affects important characteristics such as

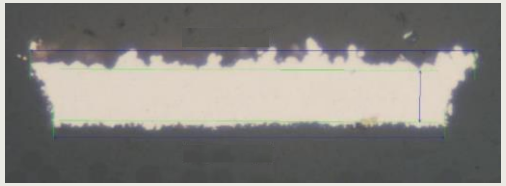
- Propagation constant
- Attenuation
- Mode profile

Need for accurate, full-wave modeling

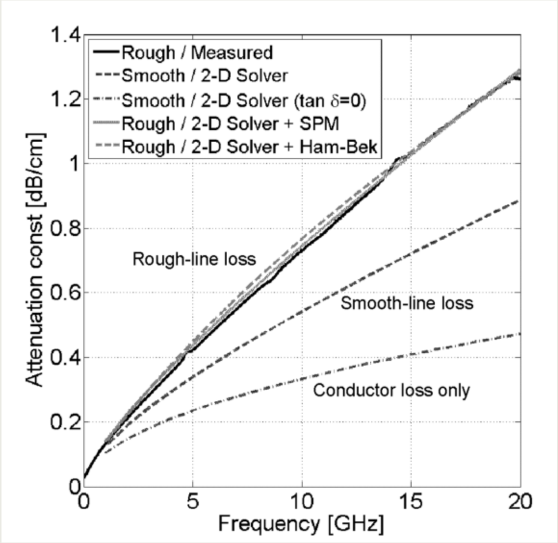
Model



Fabricated



Attenuation [1]

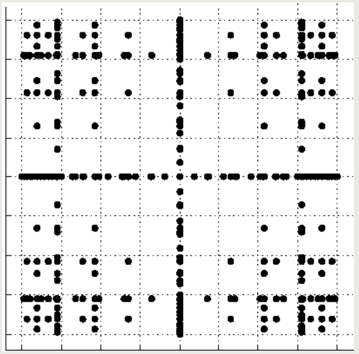


[1] H. Braunisch et al. ECTC 2014

Motivation

Line edge roughness

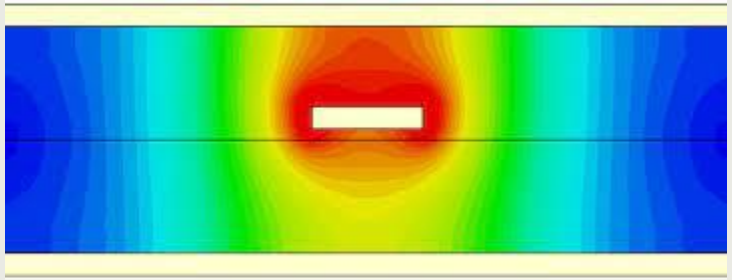
LER is a stochastic process → stochastic analysis required



Brute-force methods such as Monte Carlo (MC) require unwieldy number of runs

Sparse techniques strongly reduce calls to full-wave solver

Every call solves costly matrix problem



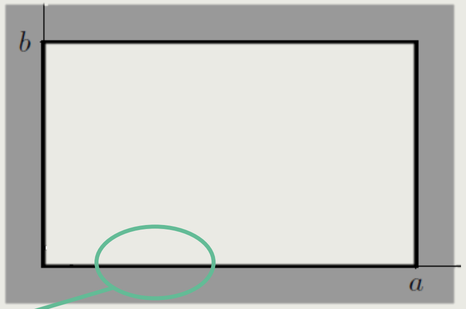
GOAL: Reduce total calculation time by combining sparse grid stochastic testing and model order reduction

Methodology

Methodology: step 1

Full-wave solver

Task: Compute propagation constant β of interconnect cross-section

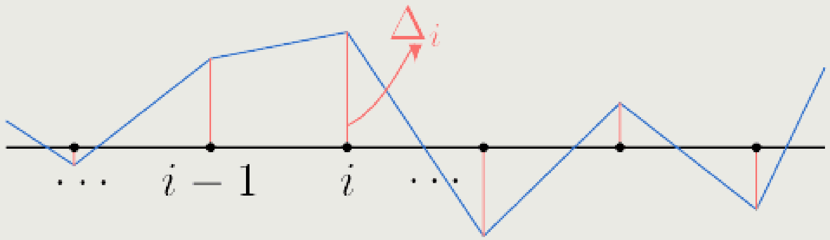


Finite element method (FEM) solver with hierarchical basis functions

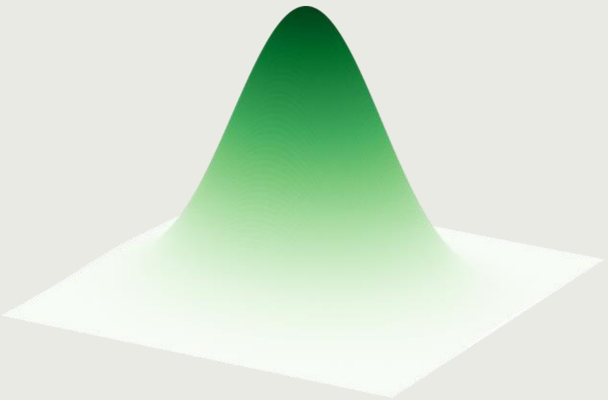
Roughness realized by displacing every node randomly according to a multivariate Gaussian distribution

zoom

n : number of basis functions



N_{RV} : number of random variables



Propagation constant β obeys quadratic eigenvalue problem $\left[\overline{\overline{M}}\beta^2 + \overline{\overline{C}}\beta + \overline{\overline{K}}\right]\mathbf{v} = 0$

Convert into a linear eigenvalue problem $\overline{\overline{A}}\mathbf{x} = \beta\overline{\overline{B}}\mathbf{x}$ of dimension $2n \times 2n$

Solve for many realizations of the waveguide, e.g., using MC => very expensive!!

Methodology: step 2

Polynomial chaos expansion (PCE) + sparse-grid stochastic testing (ST)

Propagation constant modeled by a polynomial chaos expansion

$$\beta(\boldsymbol{\xi}) \approx \sum_{k=0}^{K} a_k \phi_k(\boldsymbol{\xi})$$

ϕ_k: orthonormal polynomial
$\boldsymbol{\xi}$: random variables vector

Task: Compute expansion coefficients a_k

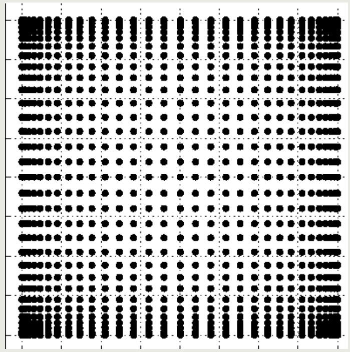
Stochastic testing: select $K+1$ *interesting* samples $\boldsymbol{\xi}_i$, compute $\beta(\boldsymbol{\xi}_i)$, and solve matrix equation

$$\begin{bmatrix} \beta(\boldsymbol{\xi}_0) \\ \vdots \\ \beta(\boldsymbol{\xi}_K) \end{bmatrix} = \begin{bmatrix} \phi_0(\boldsymbol{\xi}_0) & \dots & \phi_K(\boldsymbol{\xi}_0) \\ \vdots & \ddots & \vdots \\ \phi_0(\boldsymbol{\xi}_K) & \dots & \phi_K(\boldsymbol{\xi}_K) \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_K \end{bmatrix}$$

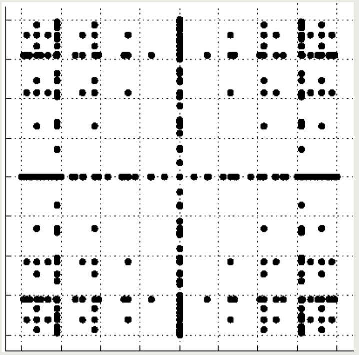
Selecting $\boldsymbol{\xi}_i$?

Node picking algorithm [2] on quadrature-based grid

Full tensor grid



Sparse (Smolyak) grid



[2] Z. Zhang et al. IEEE TCAD 2013

Methodology: step 3

Reduced-order ST

For every $\boldsymbol{\xi}_i$ (i.e., K+1 instances), the $2n \times 2n$ eigenvalue problem has to be solved:

n : number of basis functions

$$\overline{\overline{A}}\mathbf{x} = \beta\overline{\overline{B}}\mathbf{x}$$

BUT: some information is recyclable between every instance $\boldsymbol{\xi}_i$

Exploit redundancy in solving the matrix equation

Project onto smaller space through projection matrix $\overline{\overline{Q}}$ to get $m \times m$ matrix equation

$m \ll 2n$

$$\overline{\overline{A}}'\mathbf{x}' = \beta'\overline{\overline{B}}'\mathbf{x}'$$

Task: Construct $\overline{\overline{Q}}$

Base set of $N_{\mathrm{RV}} + 1$ samples $\boldsymbol{\xi}_j$: the nominal configuration $\boldsymbol{\xi}_0$ and one realization per axis of the grid (with highest weight)

Calculate the $N_{\mathrm{RV}} + 1$ full-system solutions and store corresponding eigenvectors $\mathbf{x}_j$

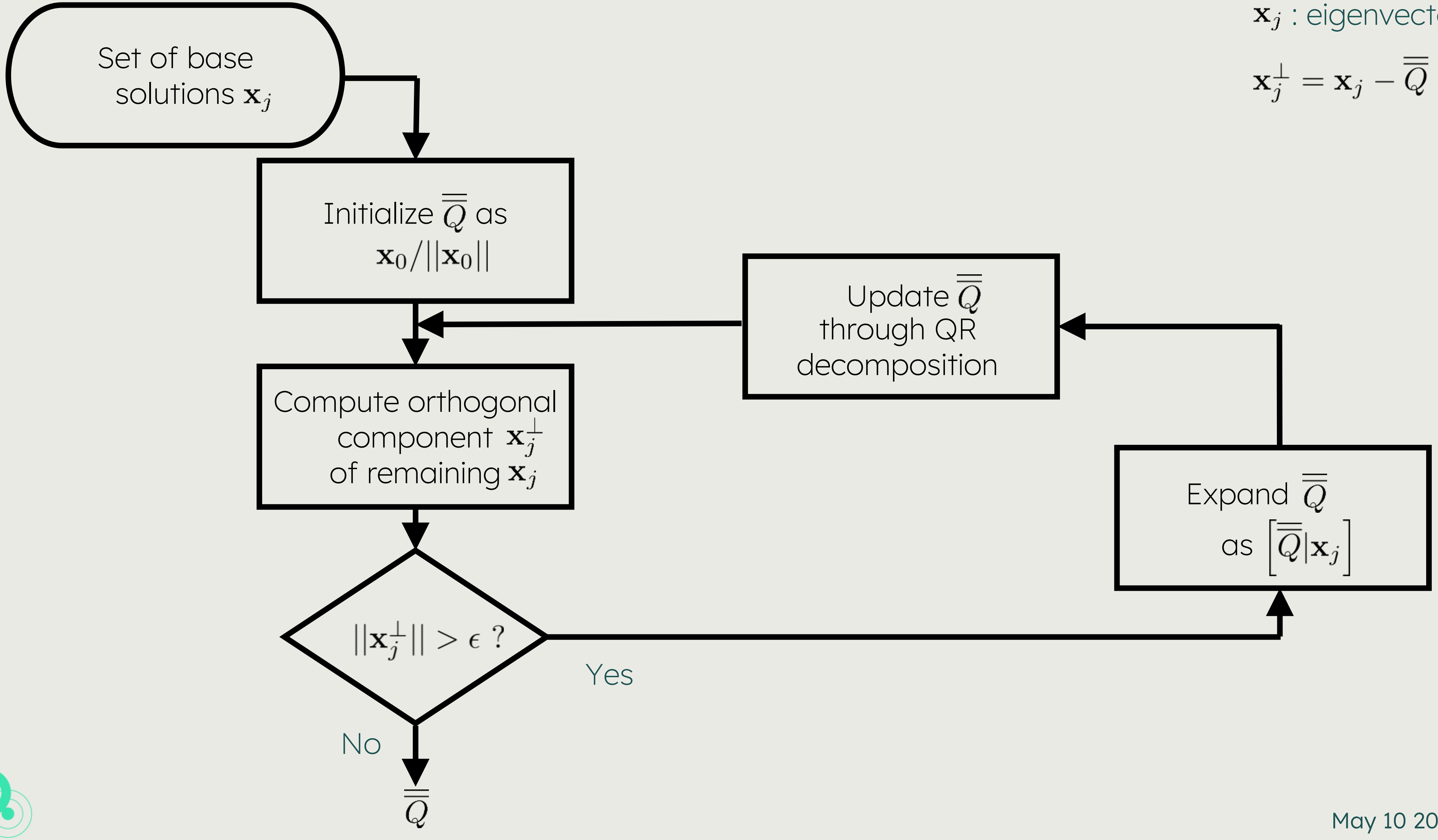
Use $\mathbf{x}_j$ to construct the orthonormal basis vectors (columns of $\overline{\overline{Q}}$)

Methodology: step 3

Constructing Q

$\mathbf{x}_j$: eigenvector

$$\mathbf{x}_j^{\perp} = \mathbf{x}_j - \overline{\overline{Q}} \left[\overline{\overline{Q}}^T \mathbf{x}_j \right]$$



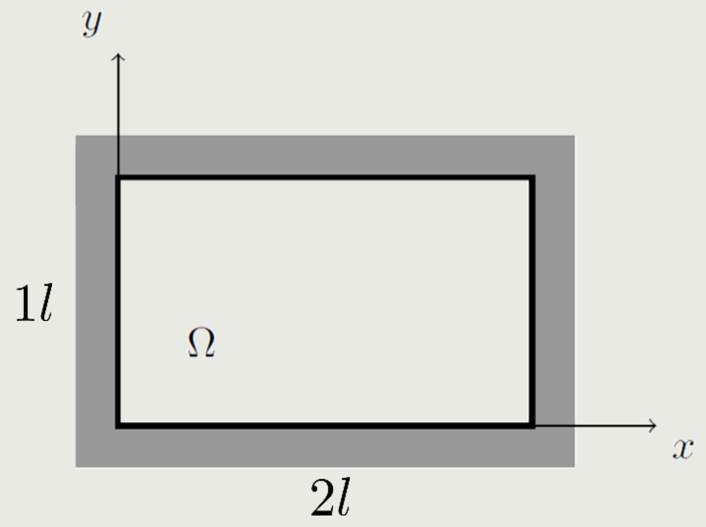
Examples

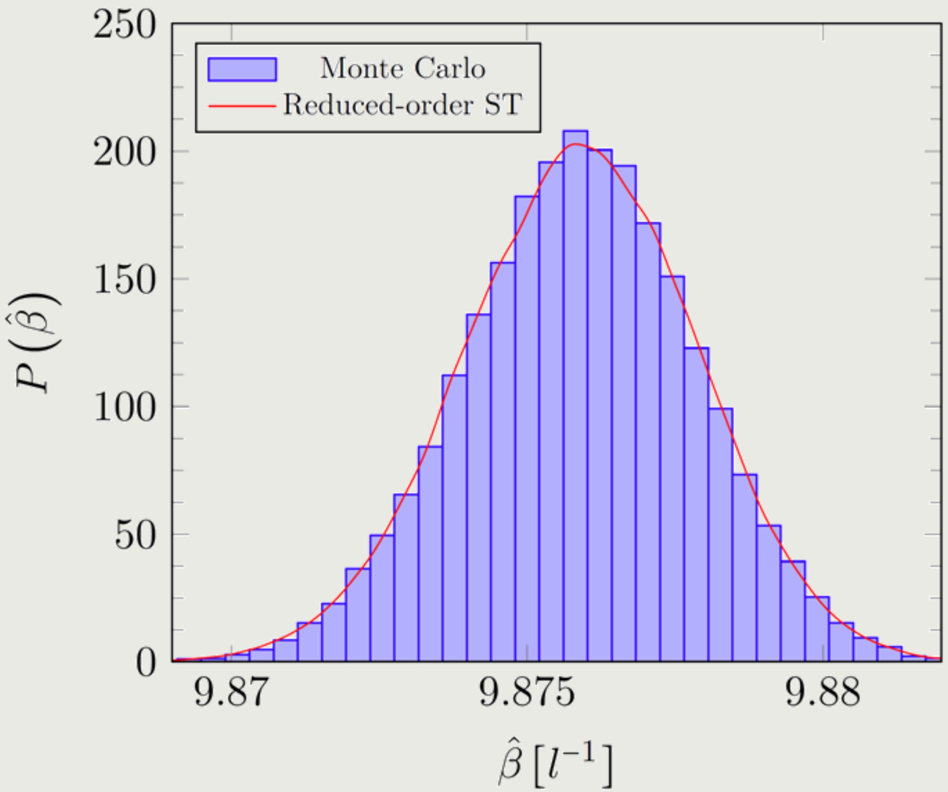
Interconnect structures

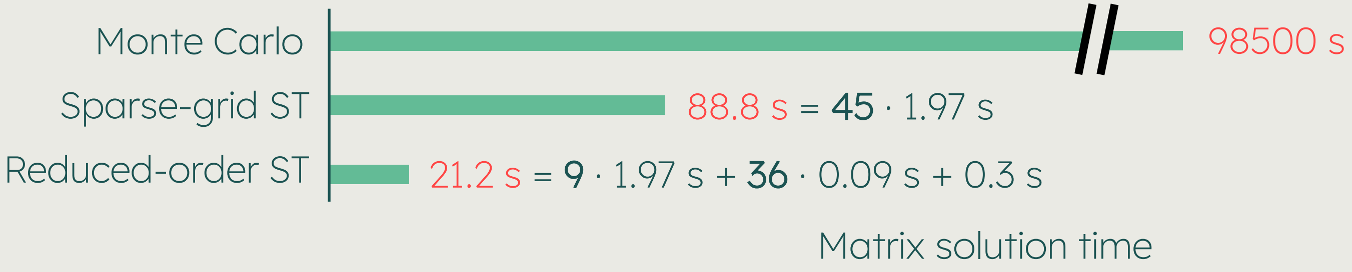
Rough rectangular waveguide

TE_{10} mode rectangular waveguide

- Correlation length: 0.5 l
- Standard deviation correlation: 0.01 l
- N_{RV} = 8
- # simulations = 45
- Wavenumber: 10 l^{-1}





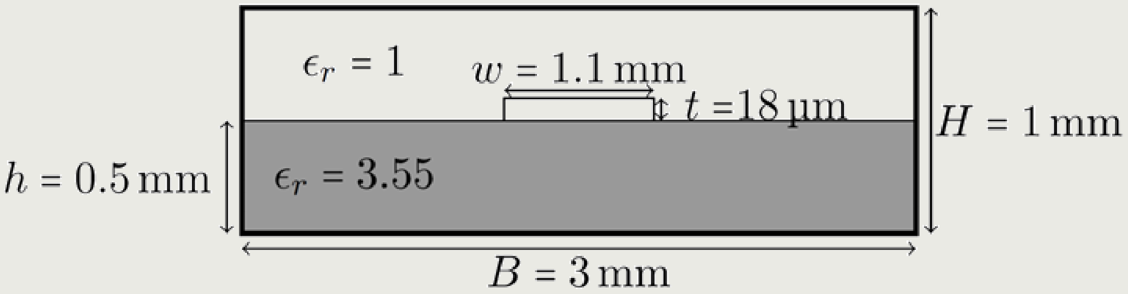


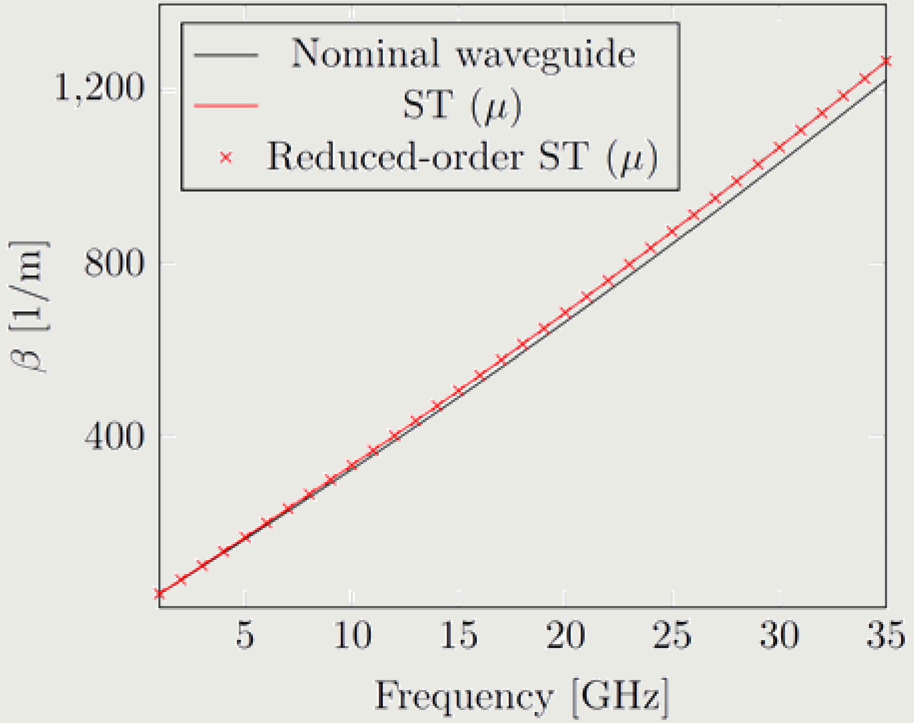
Interconnect structures

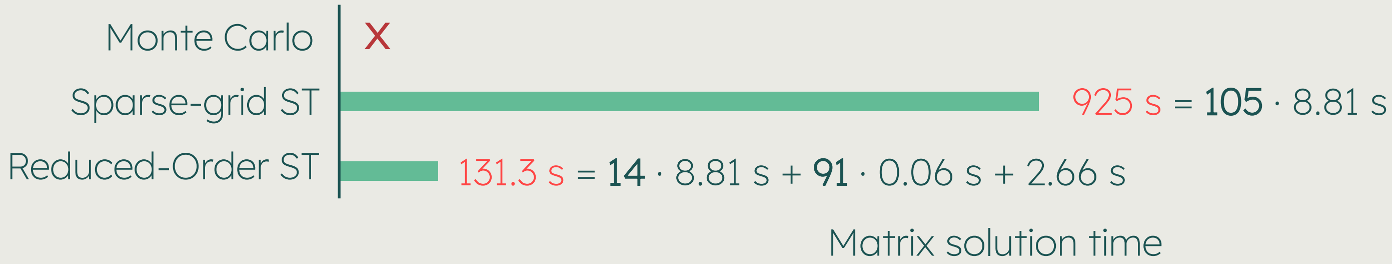
Boxed microstrip

Lowest propagating mode microstrip

- Correlation length: 0.1 mm
- Standard deviation correlation: 1 μm
- $N_{RV} = 13$
- # simulations = 105







Conclusions

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Line edge roughness' effect on interconnect propagation constant

Sparse grid stochastic testing to decrease number of full-wave calls

Order reduction to lower eigenvalue problem solution time

Applied to rectangular waveguide and microstrip interconnect quest.

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