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Early Career Paper Finalist

We3I-2

Analytic Differential Admittance Operator Solution of a Dielectric Sphere under Radial Dipole Illumination

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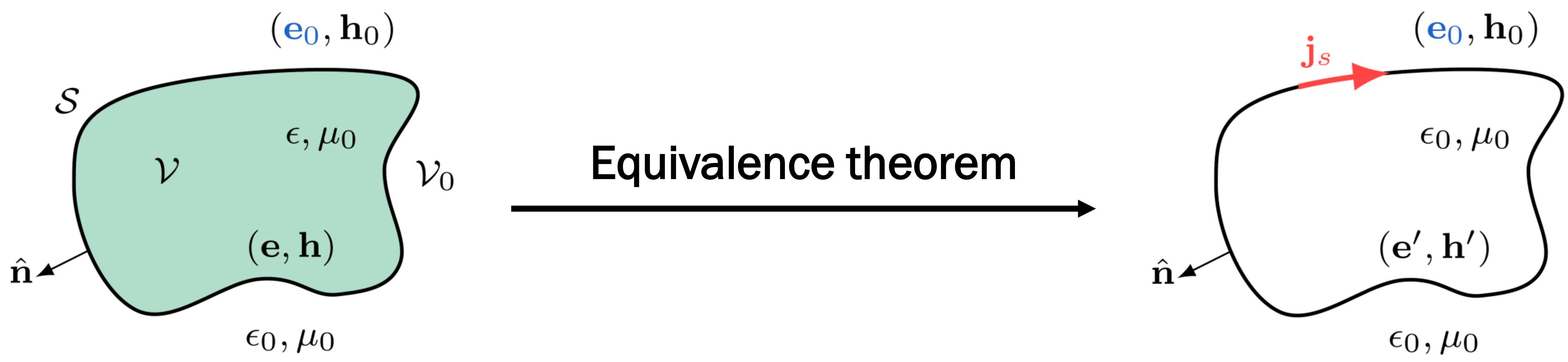
Outline

- Single-source boundary integral equations
- The differential surface admittance operator
- Spherical harmonics
- Generalized Fourier series
- Numerical analysis
- Conclusion & future work

Single-source boundary integral equation

- When applying the equivalence theorem, one can introduce 1 or 2 surface current densities
- Single source approaches sacrifice control over fields in the replaced medium for simplicity
- However, properties of such approaches are less researched



Differential surface admittance operator

- One approach is the differential surface admittance operator (DSA)
  - Has been successfully applied to
    - 2-D transmission line RLGC extraction [1]
    - Arbitrary interconnect characterization [2]
    - 3-D scattering & interconnects of canonical volumes [3]
    - Development of magnetic interconnects [4]
  - However, rigorous proof of the DSA's exactness, its inherent properties & weaknesses, and the effects of magnetic contrast is still lacking

[1] Demeester, IEEE MTT 2008
[2] Patel, IEEE MTT 2016
[3] Huynen, IEEE MTT 2020
[4] Bosman, IEEE MTT 2023

Differential surface admittance operator

- Various techniques exist to construct the electric surface current density $\mathbf{j}_s$ (see e.g. [1,2])
- The differential surface admittance (DSA) operator uses the eigenmodes of the volume to find $\mathbf{j}_s$ without relying on the Green's function:

$$\mathbf{j}_s(\mathbf{r}) = \mathcal{Y} \circ \mathbf{e}_0^t = -\eta \sum_\nu \left[\frac{\mathcal{K}_\nu}{\mathcal{N}_\nu^2} \int_\mathcal{S} (\hat{\mathbf{n}} \times \mathbf{h}_\nu^*(\mathbf{r}')) \cdot \mathbf{e}_0^t(\mathbf{r}') \, \mathrm{d}\mathbf{r}' \right] (\hat{\mathbf{n}} \times \mathbf{h}_\nu(\mathbf{r}))$$

Magnetic eigenmodes

[1] Huynen, IEEE AWPL 2016
[2] Patel, IEEE AWPL 2017

Differential surface admittance operator

- Combined with the electric field integral equation (EFIE)

$$\hat{\mathbf{n}}\times\mathbf{e}_0^t(\mathbf{r}) = \hat{\mathbf{n}}\times\mathbf{e}_{\mathrm{inc}}(\mathbf{r}) + \mathcal{T}\circ\mathbf{j}_s = \hat{\mathbf{n}}\times\mathbf{e}_{\mathrm{inc}} - j\omega\mu_0\hat{\mathbf{n}}\times\int_{\mathcal{S}}\overline{\overline{G}}_0(|\mathbf{r}-\mathbf{r}'|)\mathbf{j}_s(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

  the complete system can be solved

- But what about the properties of the DSA operator:

  low-/high-frequency breakdown, functional space mapping?

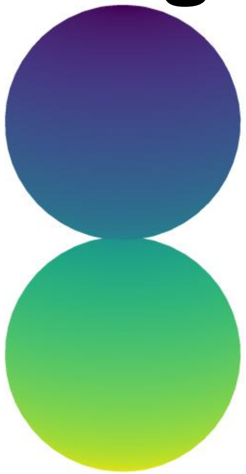
- Let us turn to the sphere to get a fully analytical solution for all operators involved

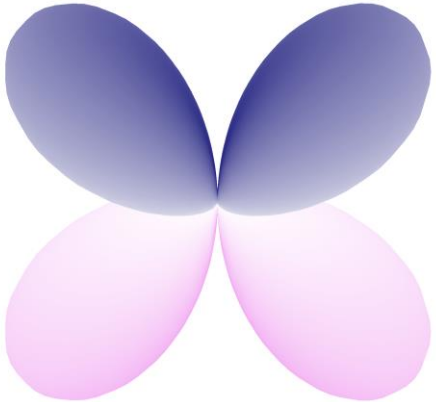
Decomposition into spherical harmonics

- We start by expanding the unknowns $\mathbf{j}_s$ and $\mathbf{e}_0^t$ on the boundary:

$$\mathbf{j}_s = \sum_{n=0}^{N} \sum_{m=-n}^{n} \alpha_{nm}^{(1)} \mathbf{u}_{nm}^{(1)} + \alpha_{nm}^{(2)} \mathbf{u}_{nm}^{(2)} \qquad \mathbf{e}_0^t = \sum_{n=0}^{N} \sum_{m=-n}^{n} \beta_{nm}^{(1)} \mathbf{u}_{nm}^{(1)} + \beta_{nm}^{(2)} \mathbf{u}_{nm}^{(2)}$$

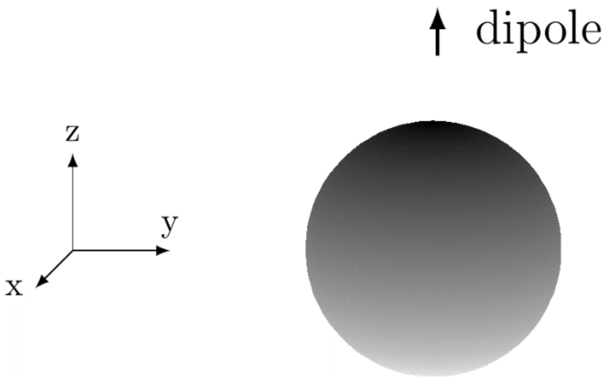
- The basis functions are two orthogonal sets of vector spherical harmonics. E.g.:

$\mathbf{u}_{nm}^{(1)} \propto$  (n,m)=(1,0)

$\mathbf{u}_{nm}^{(2)} \propto$  (n,m)=(2,-1)

Symmetry radial electric dipole

- If we consider a sphere excited by a dipole oriented along the z-axis, the results will be independent of φ (thus m=0)



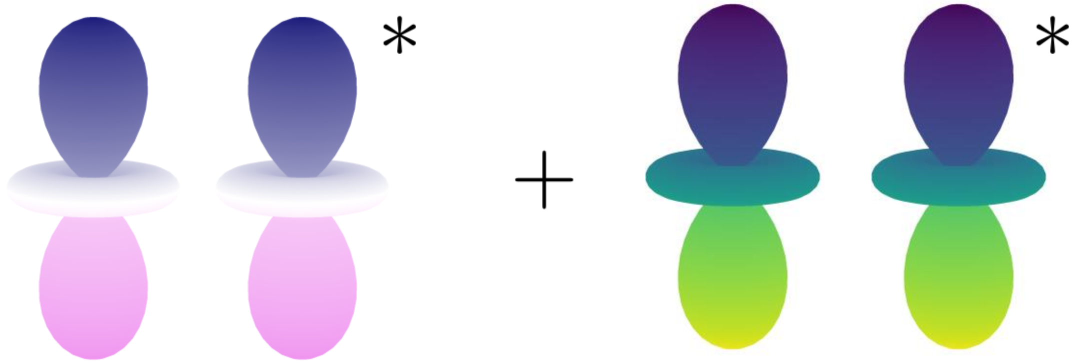
- Moreover, only the first set of basis functions is required

$$\mathbf{j}_s = \sum_{n=0}^{N} \cancel{\sum_{m=-n}^{n}} \alpha_{n0}^{(1)} \mathbf{u}_{n0}^{(1)} + \cancel{\alpha_{nm}^{(2)} \mathbf{u}_{nm}^{(2)}} \qquad \mathbf{e}_0^t = \sum_{n=0}^{N} \cancel{\sum_{m=-n}^{n}} \beta_{n0}^{(1)} \mathbf{u}_{n0}^{(1)} + \cancel{\beta_{nm}^{(2)} \mathbf{u}_{nm}^{(2)}}$$

EFIE and excitation

- The Green's dyadic & the dipole's radiation pattern can be expanded in vector spherical harmonics as well [1].

$$\overline{\overline{G}}_0 \sim \sum_{m,n} d_{nm}$$



[1] Hsiao et al. IEEE TAP 1997

EFIE and excitation

- Since these basis functions are orthogonal, testing with $\mathbf{u}_{nm}^{(1)}$ and integrating analytically, isolates a single $\alpha_n^{(1)}$,$\beta_n^{(1)}$ pair [1]

$$\hat{\mathbf{n}} \times \mathbf{e}_0^t(\mathbf{r}) = \hat{\mathbf{n}} \times \mathbf{e}_{\text{inc}} - j\omega\mu_0 \hat{\mathbf{n}} \times \int_{\mathcal{S}} \overline{\overline{G}}_0(|\mathbf{r}-\mathbf{r}'|)\mathbf{j}_s(\mathbf{r}')\,\mathrm{d}\mathbf{r}'$$

$$\mathbf{e}_0^t = \sum_{n=0}^{N} \beta_n^{(1)} \mathbf{u}_{n0}^{(1)}$$

$$\mathbf{j}_s = \sum_{n=0}^{N} \alpha_n^{(1)} \mathbf{u}_{n0}^{(1)}$$

$$\beta_n^{(1)} = \gamma_n + \mathcal{Z}_n \alpha_n^{(1)}$$

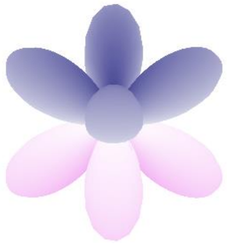
$$\mathcal{Z}_n \propto \left[k_0 a h_n^{(2)}(k_0 a)\right]' \left[k_0 a j_n(k_0 a)\right]'$$

[1] Hsiao et al. IEEE TAP 1997

Eigenmodes spherical cavity

- The magnetic eigenmodes of a spherical PEC cavity $\mathbf{h}_\nu$ are of the same form as the vector spherical harmonics $\mathbf{u}_{nm}^{(1)}$ & $\mathbf{u}_{nm}^{(2)}$:

$$\mathbf{h}_{nms} \propto r j_n(k_{ns} r)\mathbf{u}_{nm}^{(2)} \quad \& \quad \mathbf{h}_{nms} \propto r j_n(k_{ns} r)\mathbf{u}_{nm}^{(1)}$$



- The same orthogonality applies so…

Differential surface admittance

- … testing with $\mathbf{u}_{nm}^{(1)}$, isolates a single $\alpha_n^{(1)}, \beta_n^{(1)}$ pair:

$$\mathbf{j}_s(\mathbf{r}) = -\eta \sum_\nu \left[\frac{\mathcal{K}_\nu}{\mathcal{N}_\nu^2} \int_{\mathcal{S}} (\hat{\mathbf{n}} \times \mathbf{h}_\nu^*(\mathbf{r}')) \cdot \mathbf{e}_0^t(\mathbf{r}')\, \mathrm{d}\mathbf{r}' \right] (\hat{\mathbf{n}} \times \mathbf{h}_\nu(\mathbf{r}))$$

$$\mathbf{j}_s = \sum_{n=0}^{N} \alpha_n^{(1)} \mathbf{u}_{n0}^{(1)}$$

$$\mathbf{e}_0^t = \sum_{n=0}^{N} \beta_n^{(1)} \mathbf{u}_{n0}^{(1)}$$

$$\alpha_n^{(1)} = \mathcal{Y}_n \beta_n^{(1)}$$

with

$$\mathcal{Y}_n \propto \sum_{s=1}^{\infty} \frac{2k_{ns}^2 \left(k^2 - k_0^2\right)}{(k_{ns}^2 - k^2)(k_{ns}^2 - k_0^2) \left[1 - \frac{n(n+1)}{x_{ns}^2}\right]}$$

&

$$[x_{ns} j_n(x_{ns})]' = 0$$

Summary

- By expanding the unknown boundary quantities in spherical harmonics:

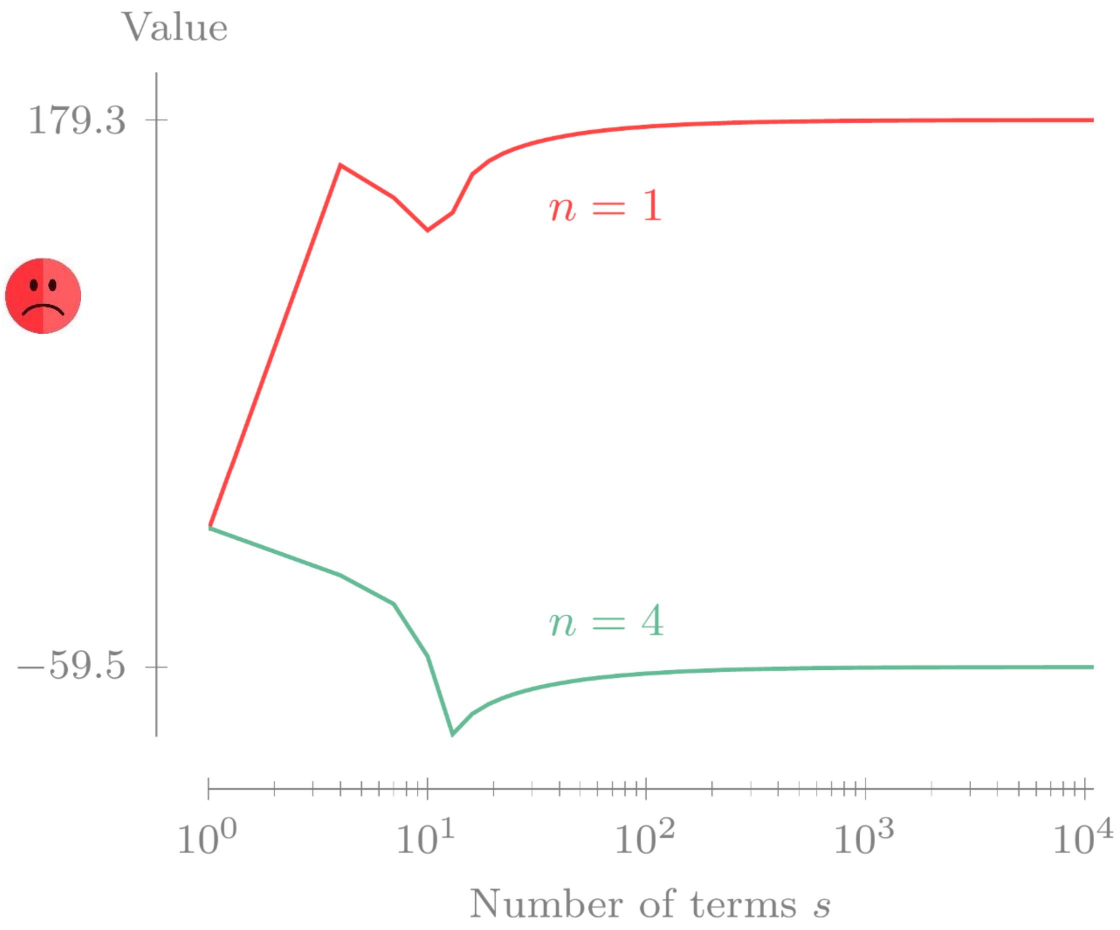
$$\mathbf{j}_s = \sum_{n=0}^{N} \alpha_n^{(1)} \mathbf{u}_{n0}^{(1)} \qquad \mathbf{e}_0^t = \sum_{n=0}^{N} \beta_n^{(1)} \mathbf{u}_{n0}^{(1)}$$

- and fully making use of the spherical harmonics expansions of the EFIE and DSA operator, we have found analytical solutions for all $\alpha_n^{(1)}$ and $\beta_n^{(1)}$ coefficients

$$\beta_n^{(1)} = \gamma_n + \mathcal{Z}_n \alpha_n^{(1)} \qquad \alpha_n^{(1)} = \mathcal{Y}_n \beta_n^{(1)}$$

Sum over the Bessel zeroes

- The factor $\mathcal{Y}_n$ contains an infinite sum that needs to be evaluated
- Evaluation requires x_{ns}, which need to be computed numerically
- A lot of terms are required to approach the asymptote
- Closed form exists?



Generalized Fourier series

- Yes, using a generalized Fourier series 🙂

$$f(r) = \sum_{s=1}^{\infty} c_s\, j_n(k_{ns} r) = \sum_{s=1}^{\infty} \frac{\langle f, j_n(k_{ns} r)\rangle}{||j_n(k_{ns} r)||^2} j_n(k_{ns} r),$$

- This function f results in a similar sum

$$f(r) = \frac{(ka)^2}{[ka j_n(ka)]'} j_n(kr),$$

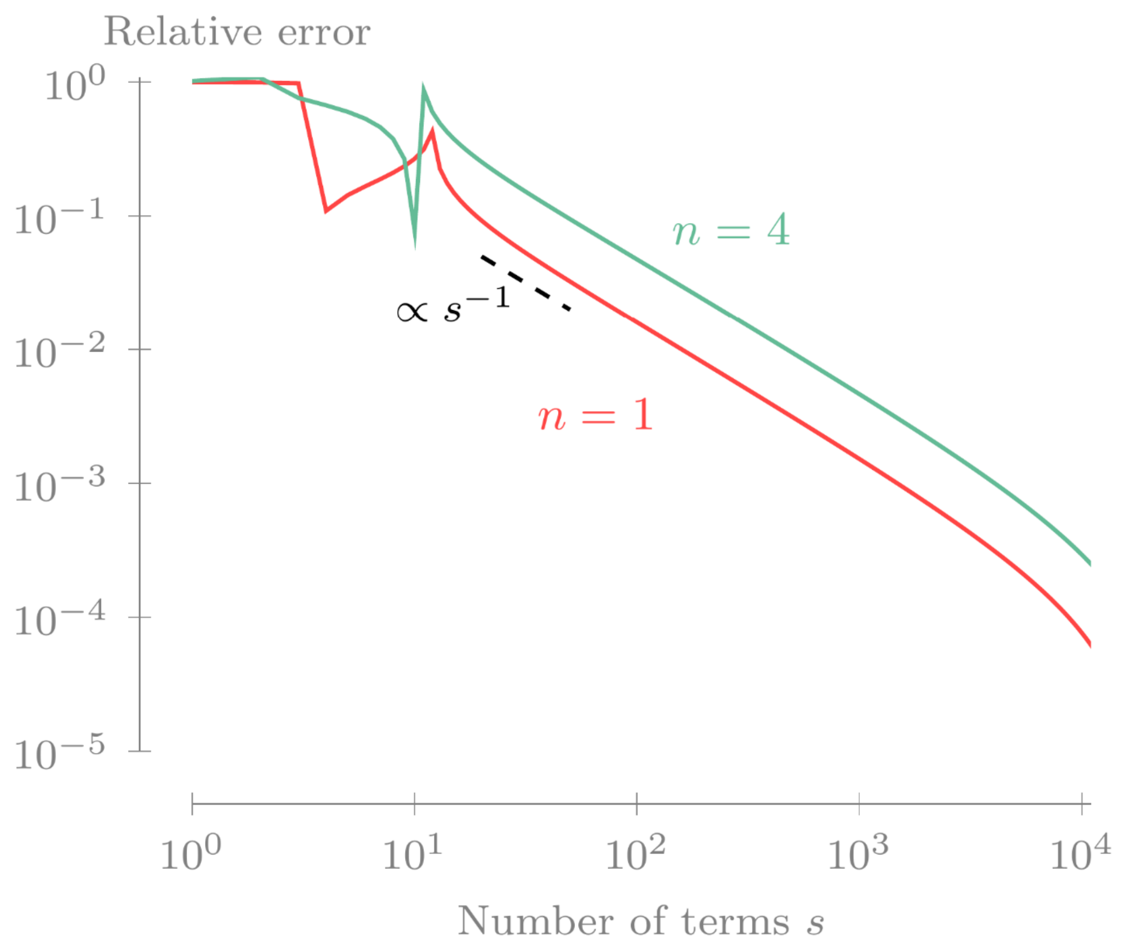
- Closed form contains just a few function evaluations 🙂

$$\mathcal{Y}_n \propto \sum_{s=1}^{\infty} \frac{2k_{ns}^2\left(k^2 - k_0^2\right)}{(k_{ns}^2 - k^2)(k_{ns}^2 - k_0^2)\left[1 - \frac{n(n+1)}{x_{ns}^2}\right]}$$

$$\mathcal{Y}_n \propto \frac{(ka)^2 j_n(ka)}{[ka j_n(ka)]'} - \frac{(k_0 a)^2 j_n(k_0 a)}{[k_0 a j_n(k_0 a)]'}$$

Closed form: convergence

- Relative error of the series compared to the closed sum for n=1 & 4
- Sum converges very slowly ☹
- Analytical expression provides tremendous speed-up & accuracy improvement of the complete solution ☺



Final field coefficients

- With the $\mathcal{Y}_n$ sum issue solved, the coefficients for the electric field $\beta_n^{(1)}$ are fully defined:

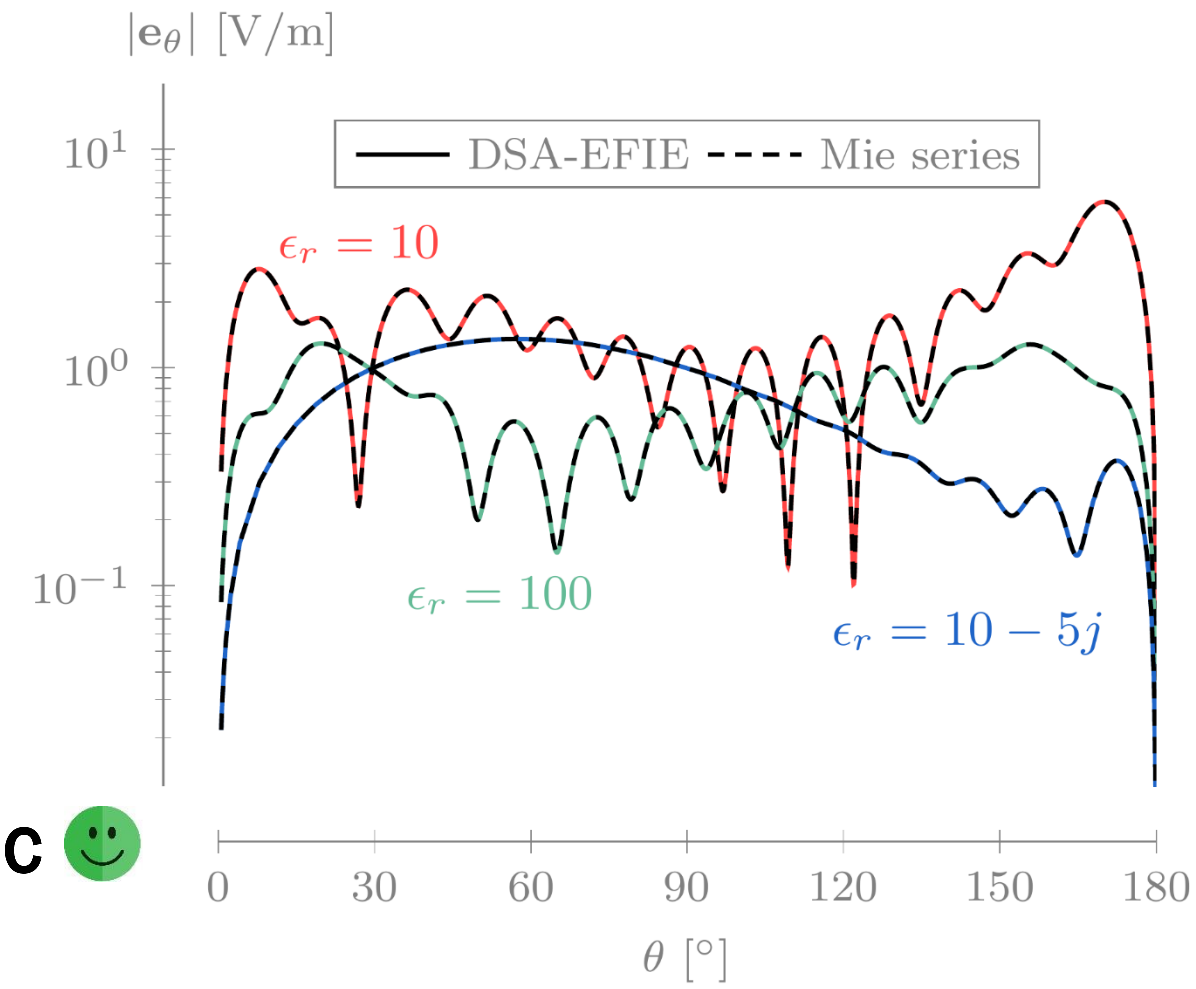
$$\alpha_n^{(1)} = \mathcal{Y}_n \gamma_n / (1 - \mathcal{Z}_n \mathcal{Y}_n) \qquad \beta_n^{(1)} = \gamma_n / (1 - \mathcal{Z}_n \mathcal{Y}_n)$$

- With the unknown coefficients fully computed, we can reconstruct the total fields

Numerical analysis

- Sphere with radius 1 m
- Dipole at 10 m on z-axis
- Tangential electric field for k=4π/1m
- # terms in series = 50

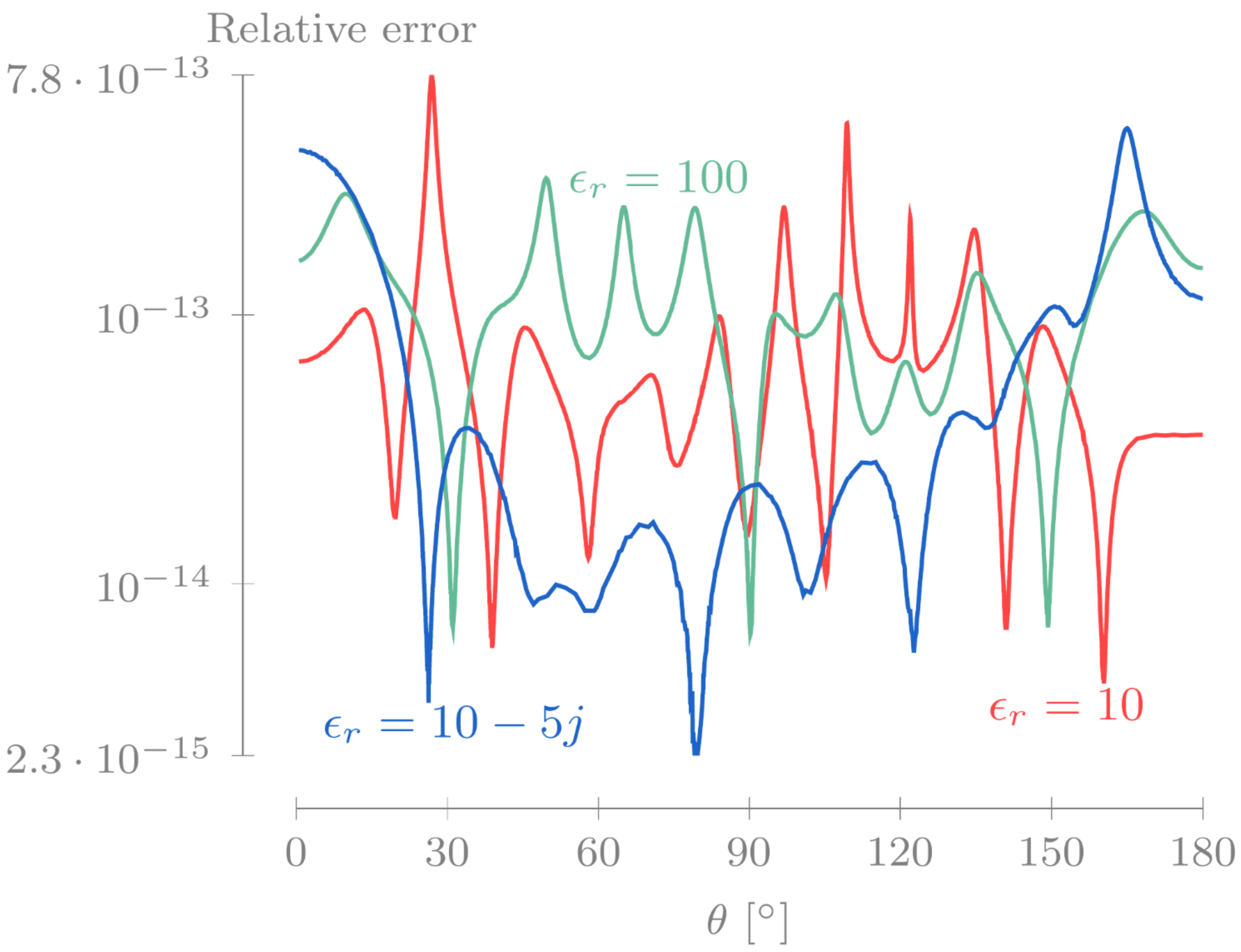
- Excellent agreement with Mie series for lossy, low- and high- contrast dielectric 🙂



Numerical analysis

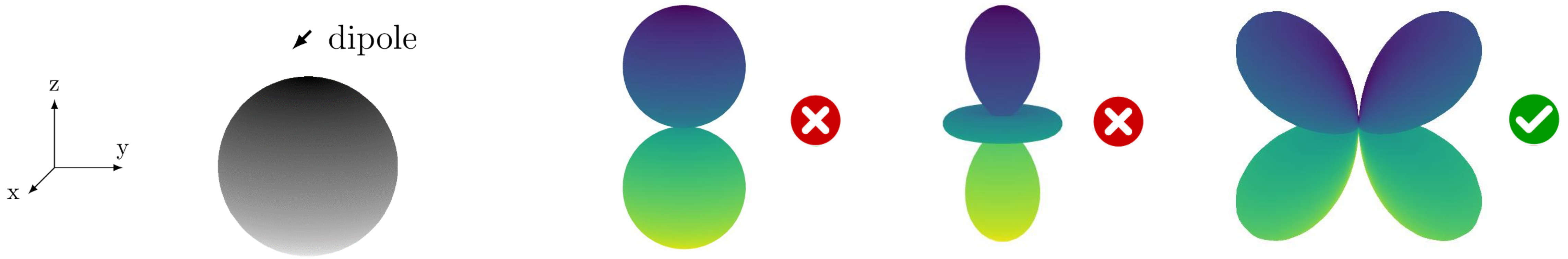
- Sphere with radius 1 m
- Dipole at 10 m on z-axis
- Tangential electric field for k=4π/1m
- # terms in series = 50

- Match of at least 12 significant digits 🙂



Symmetry tangential electric dipole

- If we consider a sphere excited by a dipole oriented along the x-axis, the results have a sin/cos(φ) (thus m=-1/1) dependency



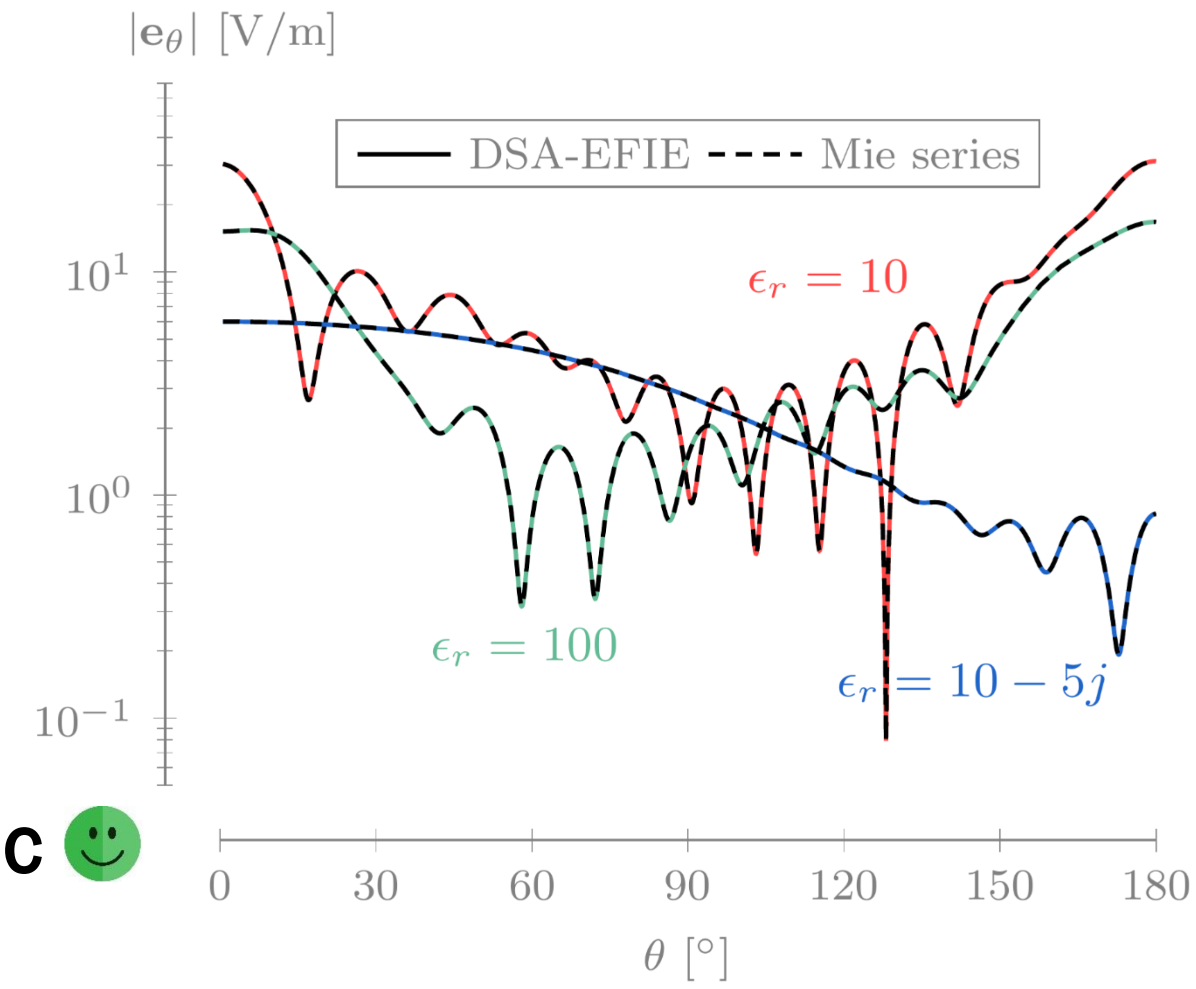
- Now, the two sets of basis functions are required

$$\mathbf{j}_s = \sum_{n=0}^{N} \sum_{m=-n}^{n} \alpha_{nm}^{(1)} \mathbf{u}_{nm}^{(1)} + \alpha_{nm}^{(2)} \mathbf{u}_{nm}^{(2)} \qquad \mathbf{e}_0^t = \sum_{n=0}^{N} \sum_{m=-n}^{n} \beta_{nm}^{(1)} \mathbf{u}_{nm}^{(1)} + \beta_{nm}^{(2)} \mathbf{u}_{nm}^{(2)}$$

Numerical analysis

- Sphere with radius 1 m
- Dipole at 10 m on x-axis
- Tangential electric field (θ) for k=4π/1m
- # terms in series = 50

- Excellent agreement with Mie series for lossy, low- and high- contrast dielectric 🙂



Numerical analysis

- Sphere with radius 1 m
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- Tangential electric field (φ) for k=4π/1m
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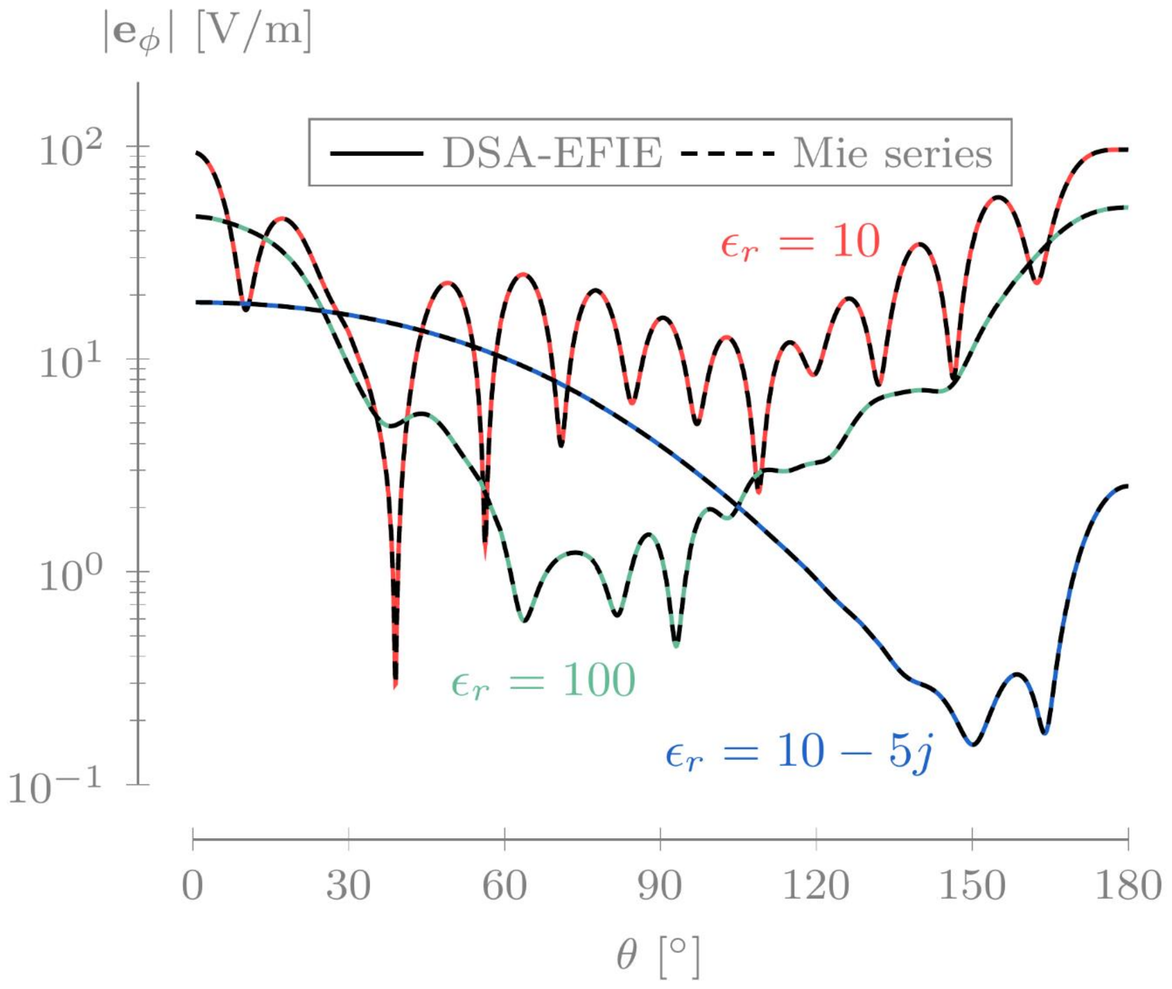
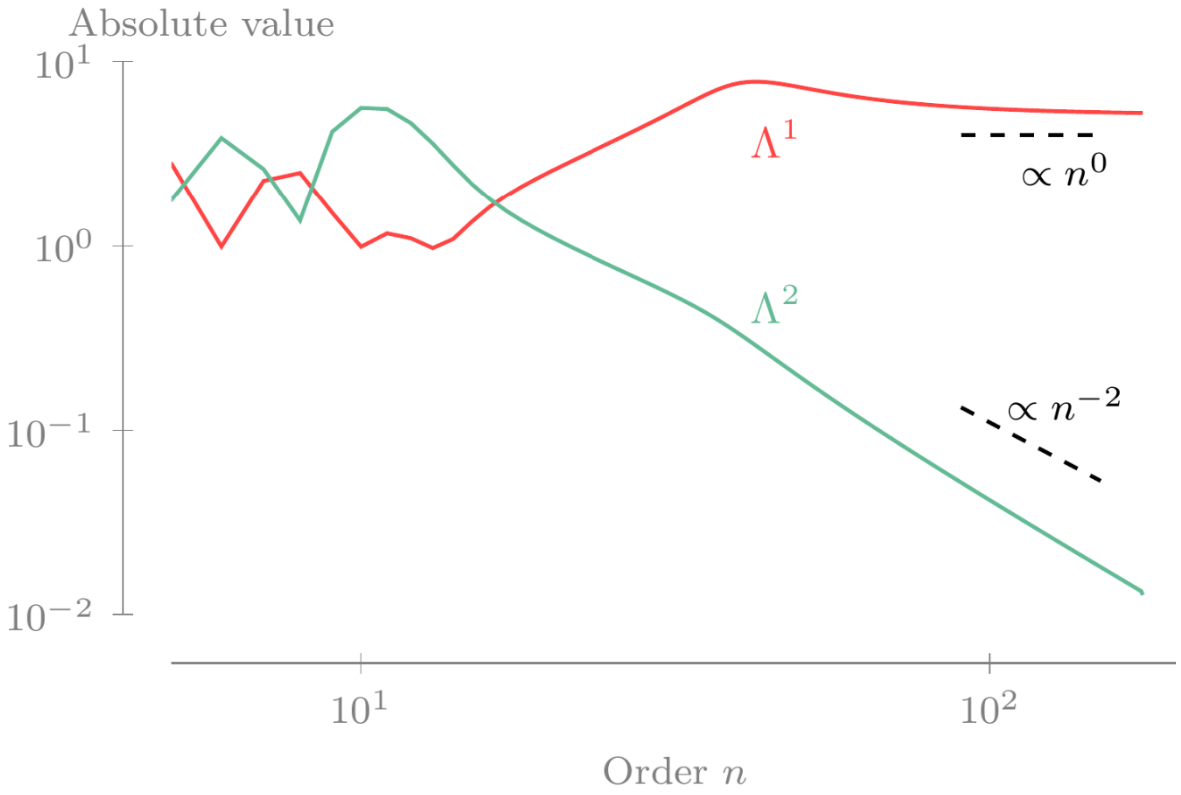
- Match of at least 12 significant digits 🙂



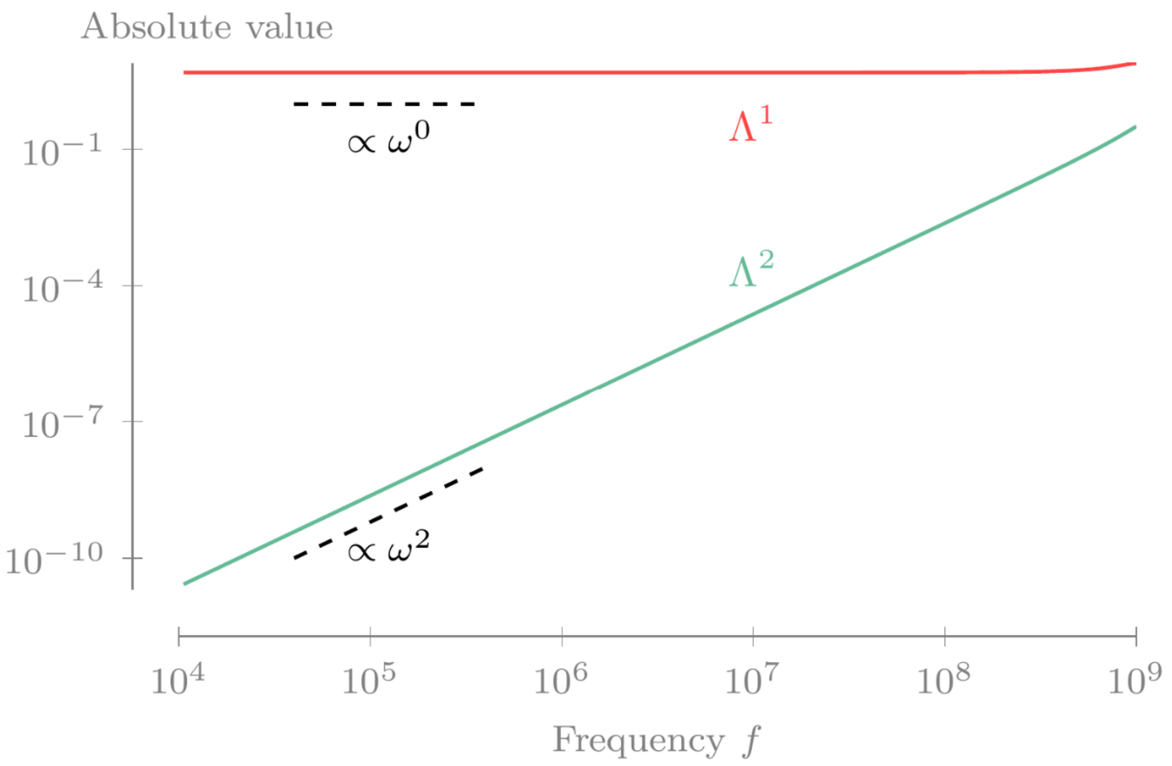
Dense mesh breakdown

- Complete DSA-EFIE system has two sets of eigenmodes
- For large n, the red set suffers from dense mesh breakdown; the green set does not (different asymptotes)
- Correct choice of Sobolev testing space can solve this issue



Low-frequency breakdown

- Complete DSA-EFIE system has two sets of eigenmodes
- For small f, the two sets diverge which leads to low-frequency breakdown
- Caused by EFIE and can thus be solved with preconditioner or augmented EFIE



Conclusion & Future work

- We presented an analytical solution of the DSA-EFIE including a closed sum for the infinite series
- Improved convergence leads to a 12-digit precision
- Investigated the total system's spectrum

Future work

- Determine impact test function space
- Extension to magnetic DSA operator