Quantum
Mechanical &
Electromagnetic
Systems
Modelling Lab

A Novel Vectorial Unified Transform for the Full-Wave Broadband Characterization of On-chip Passives

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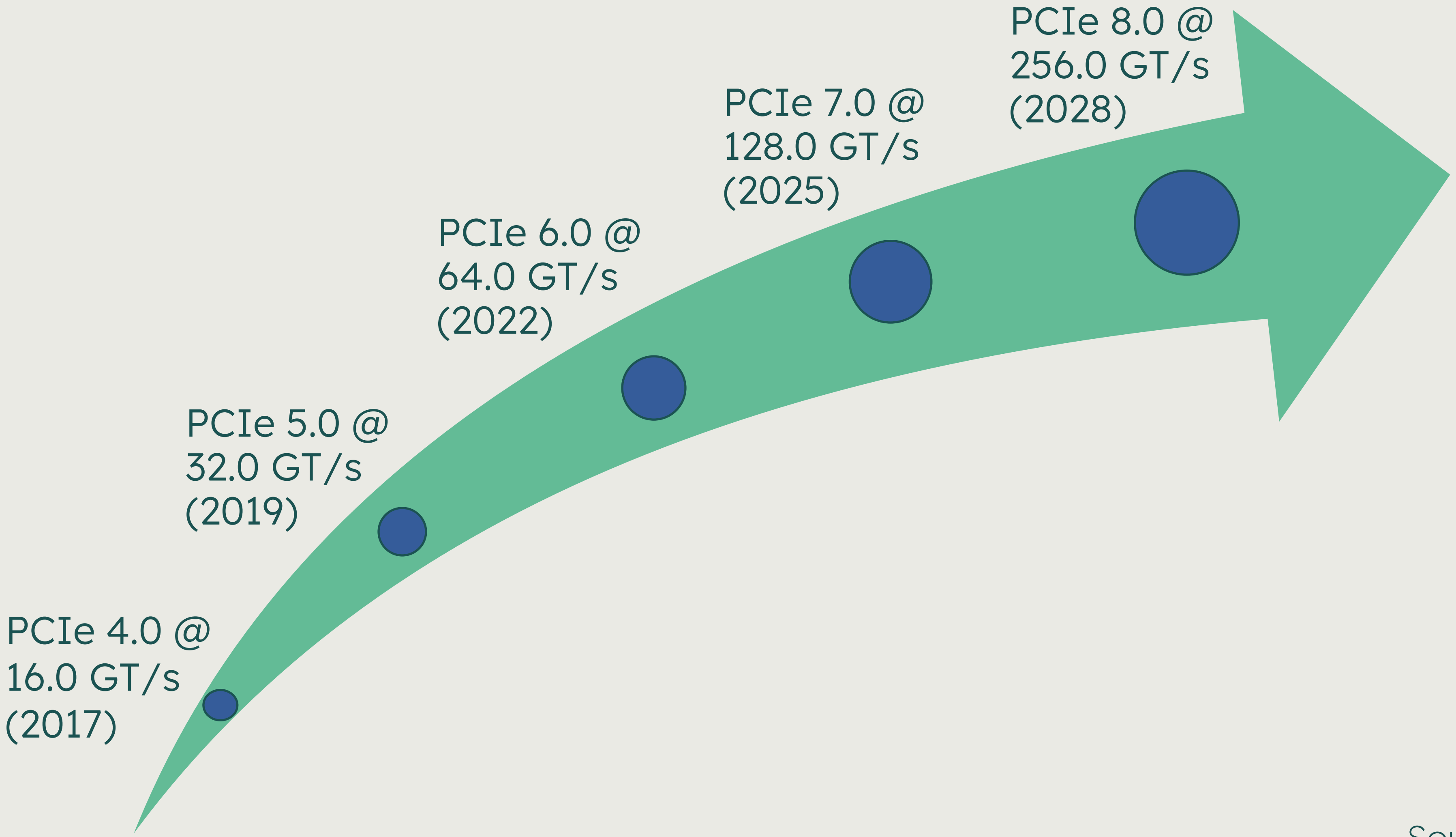
quest.

Ever-increasing trend towards higher circuit speeds

Evolution of PCI Express data rate



Source: PCI-SIG, 2025

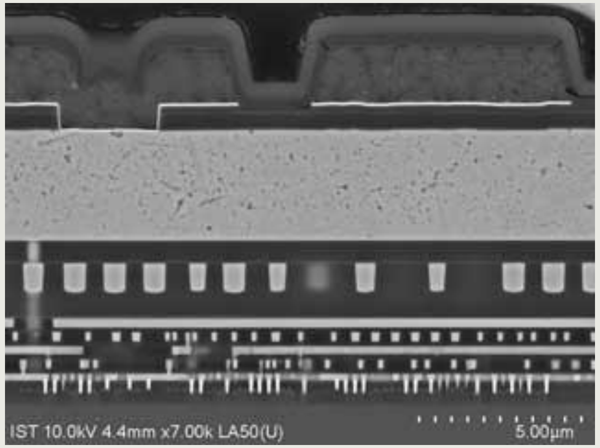
This trend requires accurate modeling

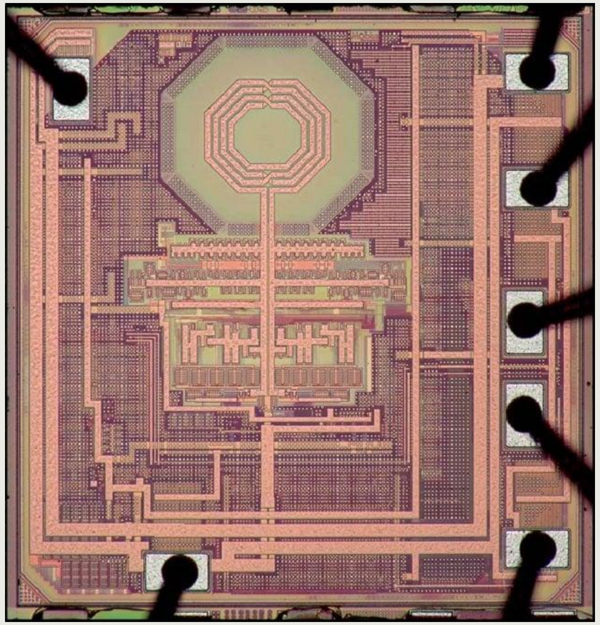
Full-wave effects become fully apparent at higher frequencies, so traditional circuit simulations no longer suffice

Heterogeneous material properties of state-of-the-art components demand precise modeling to accurately capture all electromagnetic effects

Wide variety of modeling techniques already available: FEM, VIE, PEEC, PMCHWT, GIBC, …, but each has its own benefits and drawbacks

Differential surface admittance (DSA) formalism provides an elegant **BEM** framework that allows for efficient and accurate modeling, in particular broadband characterization of good conductors





At quest, we developed a novel method for constructing the 3-D DSA operator

Several techniques already available for 3-D topologies

However, existing 3-D DSA derivation methods limited to canonical shapes or rely on inner region Green's function

Leverage unified transform framework to alleviate both these limitations

Accurately construct DSA operator for non-canonical shapes without inner region Green's function

Dirichlet-to-Neumann operator using the unified transform

DSA-EFIE formulation

Numerical results

Conclusions

Dirichlet-to-Neumann operator using the unified transform

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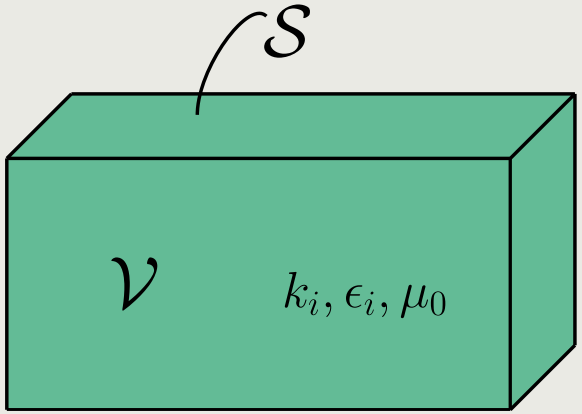
Dirichlet-to-Neumann operator using the unified transform

Dirichlet-to-Neumann (DtN) operator relates the tangential electric field on $\mathcal{S}$ with the tangential magnetic field on $\mathcal{S}$



In this work, DtN operator is derived using the unified transform (UT) method

Governing PDE (underlying physics) → (Discretized) global relation → Test solutions → DtN operator

Underlying physics governed by Maxwell's equations

Governing PDE
(underlying physics)

(Discretized)
global relation

Test solutions

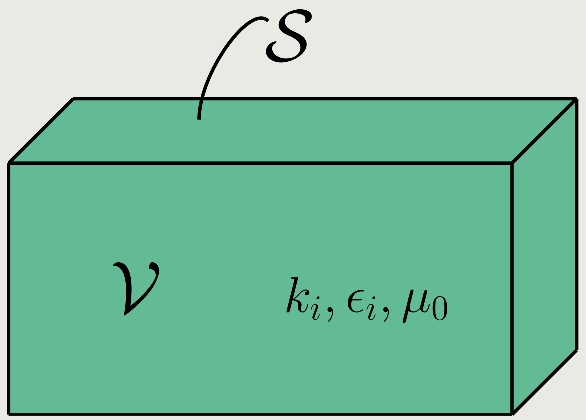
DtN operator

Sourceless Maxwell equations inside $\mathcal{V}$
Unknown solution $(\mathbf{e_a}, \mathbf{h_a})$

$$\nabla \times \mathbf{e_a} = -j\omega\mu_0 \mathbf{h_a}$$
$$\nabla \times \mathbf{h_a} = -j\omega\epsilon \mathbf{e_a}$$

Known test solution $(\mathbf{e_t}, \mathbf{h_t})$

$$\nabla \times \mathbf{e_t} = -j\omega\mu_0 \mathbf{h_t}$$
$$\nabla \times \mathbf{h_t} = j\omega\epsilon \mathbf{e_t}$$



Sought-after DtN operator relates tangential components of $\mathbf{e_a}$ and $\mathbf{h_a}$ on $\mathcal{S}$

Lorentz reciprocity as the global relation

Governing PDE
(underlying physics)

(Discretized)
global relation

Test solutions

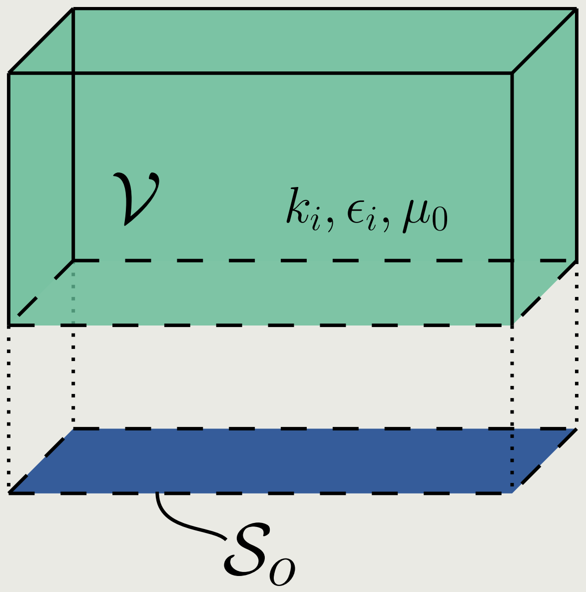
DtN operator

From known and unknown solution, we derive the global relation (= Lorentz reciprocity):

$$\iint_{\mathcal{S}} (\mathbf{e_a} \cdot \hat{\mathbf{h}}_\mathbf{t} - \mathbf{e_t} \cdot \hat{\mathbf{h}}_\mathbf{a}) dS = 0$$

For numerical evaluation, the unknown solution is discretized using Legendre polynomial-based basis functions, e.g., on $\mathcal{S}_\mathrm{o}$

$$\sum_{n,m} \iint_{\mathcal{S}_\mathrm{o}} \left(\left(e_{nm}^{o,x} \mathbf{u_x} + e_{nm}^{o,y} \mathbf{u_x} \right) \cdot \hat{\mathbf{h}}_\mathbf{t} \right) f_{nm}(x - x_o, y - y_o) dS$$
$$- \sum_{n',m'} \iint_{\mathcal{S}_\mathrm{o}} \left(\mathbf{e_t} \cdot \left(\hat{h}_{n'm'}^{o,x} \mathbf{u_x} + \hat{h}_{n'm'}^{o,y} \mathbf{u_y} \right) \right) f_{n'm'}(x - x_o, y - y_o) dS$$



Test global relation using plane waves

Governing PDE (underlying physics)

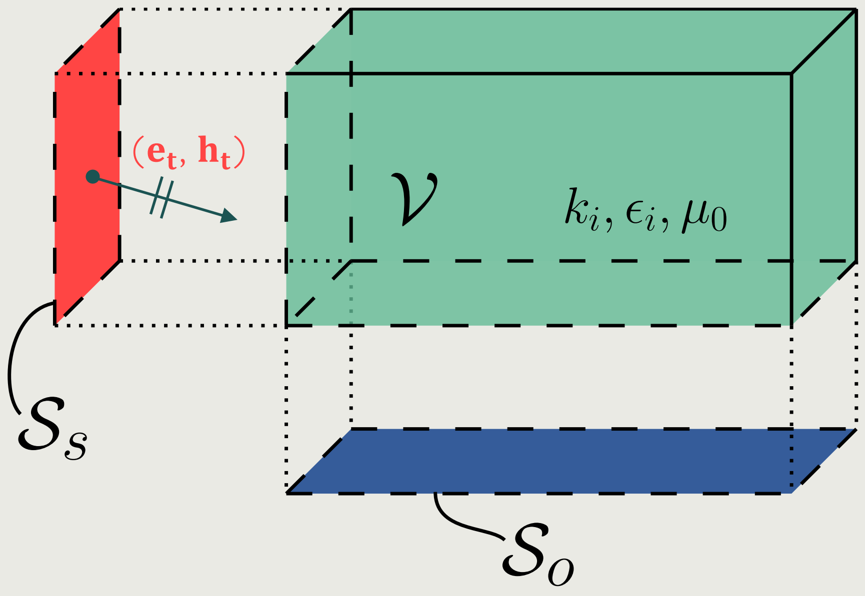
(Discretized) global relation

Test solutions

DtN operator

Evaluate discretized global relation using two orthogonal sets of plane waves originating from $\mathcal{S}_s$ as test solution $(\mathbf{e_t}, \mathbf{h_t})$

$$\text{set 1:} \quad \mathbf{e}_\mathbf{t}^1 = \mathbf{p_1} e^{-j\phi}, \ \hat{\mathbf{h}}_\mathbf{1} = (\mathbf{u} \times \mathbf{p_1}) e^{-j\phi}$$
$$\text{set 2:} \quad \mathbf{e}_\mathbf{t}^2 = \mathbf{p_2} e^{-j\phi}, \ \hat{\mathbf{h}}_\mathbf{2} = (\mathbf{u} \times \mathbf{p_2}) e^{-j\phi}$$



For a single wave of set 1, this results in

$$\sum_{n,m} \left(\frac{k_x k_y}{k\tau} e_{nm}^{o,x} + \frac{\tau}{k} e_{nm}^{o,y} \right) A_{nm}^{so} - \sum_{n\prime,m\prime} \frac{k_z}{\tau} \hat{h}_{n\prime m\prime}^{o,y} A_{n\prime m\prime}^{so}$$

Due to Fourier-type sampling of the two sets of plane waves, a total of $2 \times (2M_{\text{max}} + 1)(2P_{\text{max}} + 1)$ equations is obtained

Evaluate global relation for all cuboid faces

Governing PDE
(underlying physics)

(Discretized)
global relation

Test solutions

DtN operator

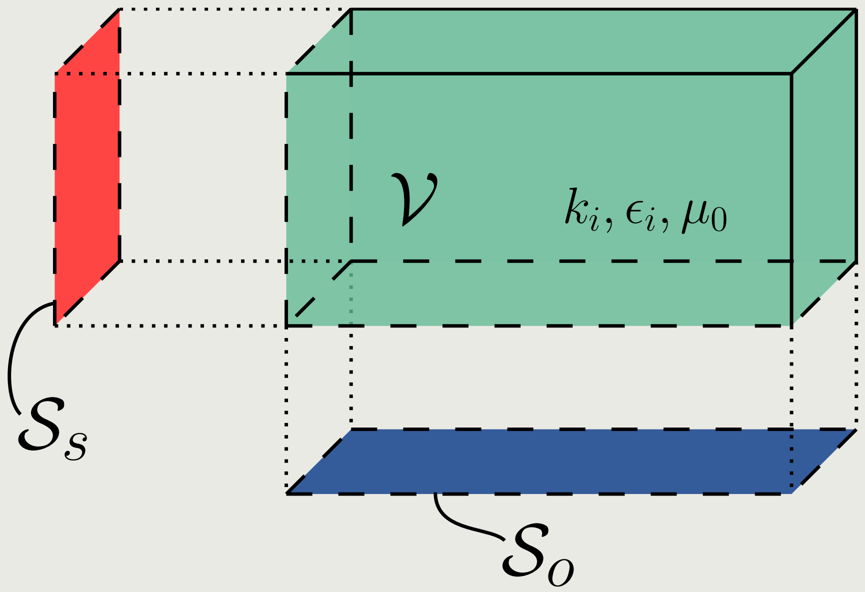
Performing this procedure for all combinations of source and observer face and collecting all equations results in

$$\bar{\bar{D}} \cdot \bar{\mathbf{e}} - \bar{\bar{N}} \cdot \bar{\hat{\mathbf{h}}} = 0$$

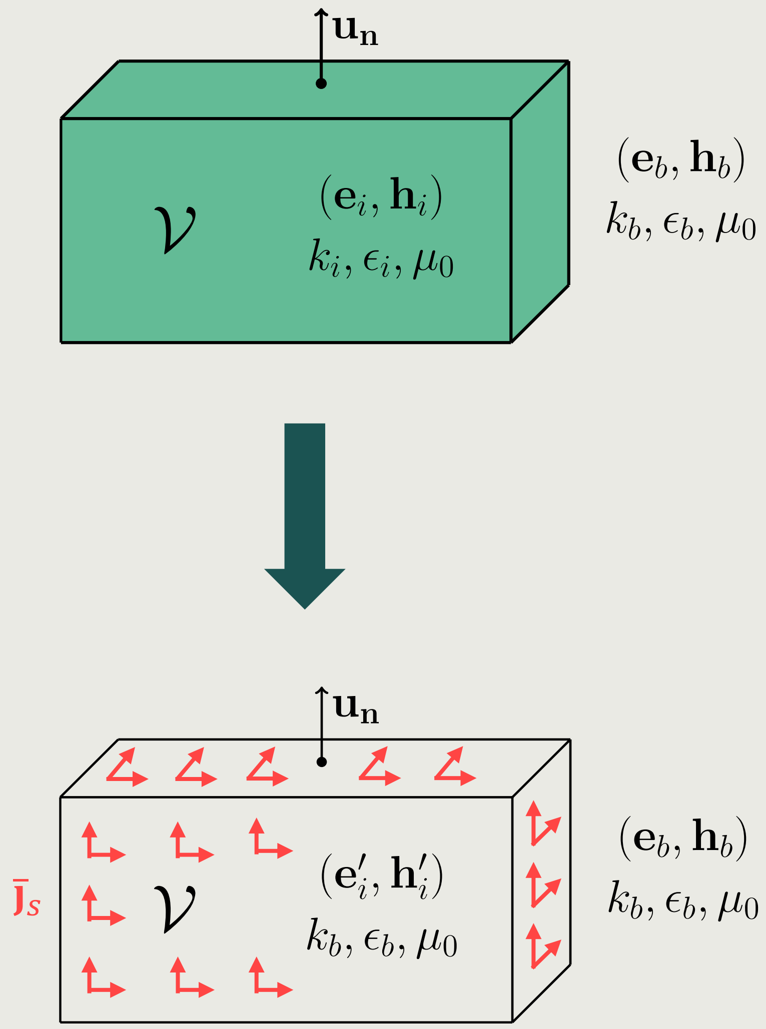
Solving this in a least-squares sense yields the sought-after discretized DtN operator $\bar{\bar{\mathcal{P}}}$

$$\bar{\mathbf{h}} = \bar{\bar{\mathcal{P}}} \cdot \bar{\mathbf{e}}$$

No eigenmodes of $\mathcal{V}$ or inner region Green's function required to construct $\bar{\bar{\mathcal{P}}}$



Construction of discretized DSA operator



The equivalence principle replaces materials by the background medium

$$\bar{\mathbf{j}}_s = \bar{\mathbf{h}}_i - \bar{\mathbf{h}}_i' = \left(\bar{\bar{\mathcal{P}}} - \bar{\bar{\mathcal{P}}}_0\right) \cdot \bar{\mathbf{e}}_\mathrm{b} = \bar{\bar{\mathcal{Y}}} \cdot \bar{\mathbf{e}}_\mathrm{b} \qquad \text{on } \mathcal{S}$$

DSA operator $\bar{\bar{\mathcal{Y}}}$ is constructed as difference of two DtN operators $\bar{\bar{\mathcal{P}}}$ and $\bar{\bar{\mathcal{P}}}_0$ for the complete volume

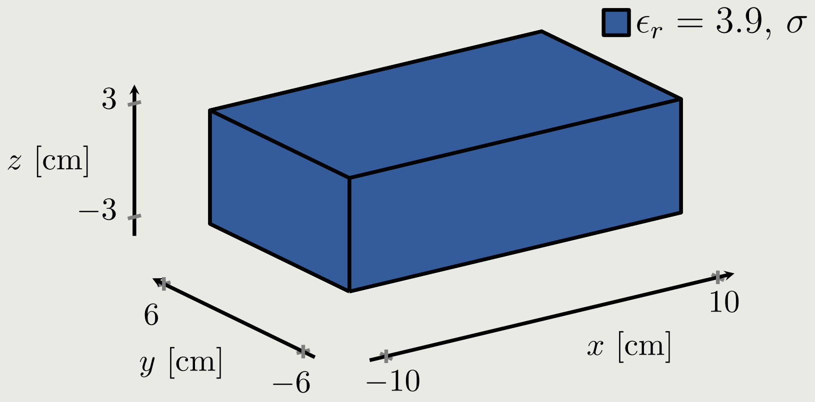
Now, only current sources in background medium → easier to solve numerically

Projecting DSA operator $\bar{\bar{\mathcal{Y}}}$ onto rooftop functions and integration with the Augmented Electric Field Integral Equation [1] provides full characterization of the electromagnetic problem

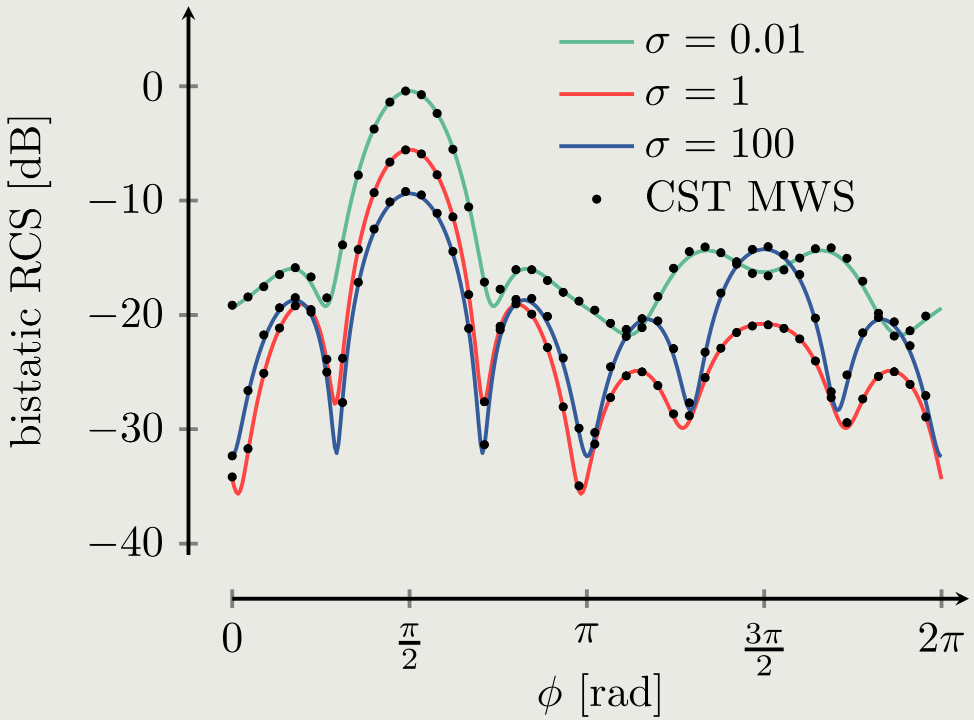
[1] Z.-G. Qian and W. C. Chew, IEEE Trans. Antennas Propag., vol. 57, no. 11, pp. 3594–3601, 2009.

Numerical results

Dielectric cuboid with varying electrical conductivity



Illuminated by plane wave propagating in **+y-direction** with a **z-polarized** electric field



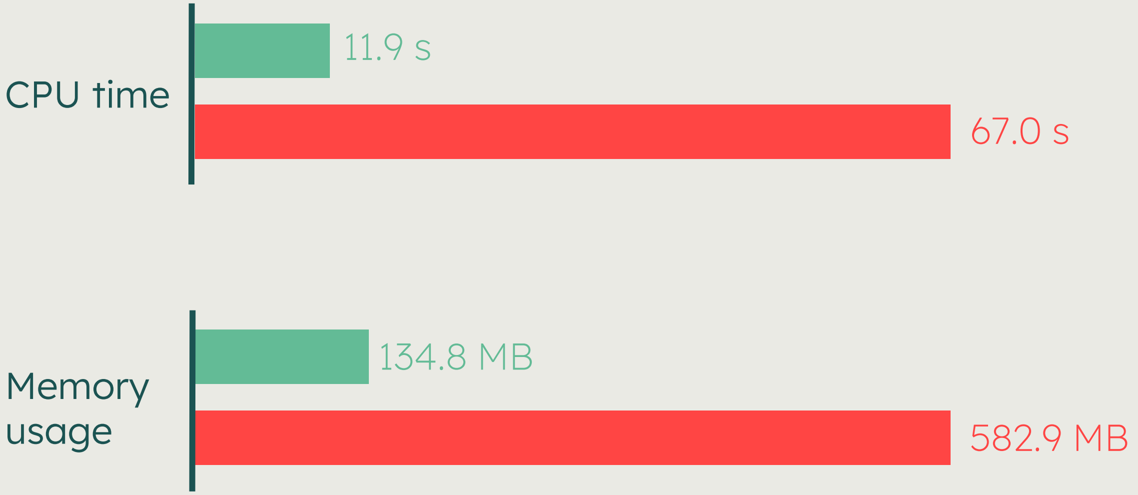
Very good agreement with CST MWS reference solution

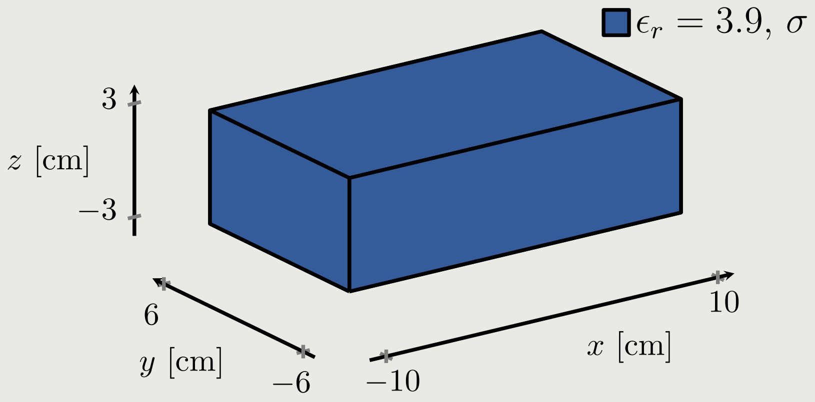
Accuracy is maintained across wide range of material parameters

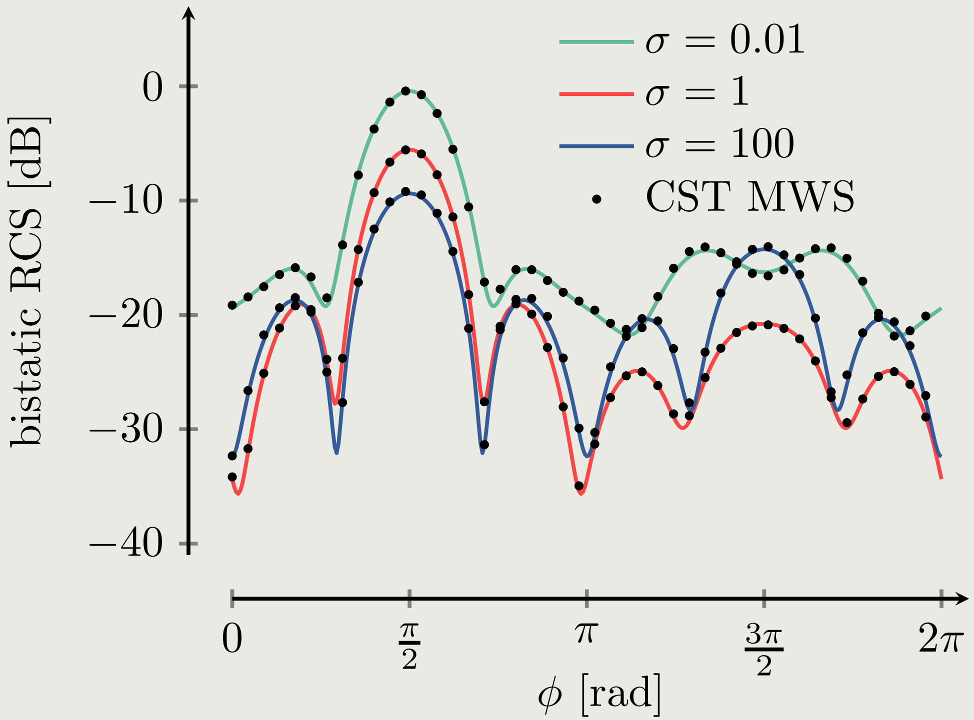
Numerical results

Dielectric cuboid with varying electrical conductivity

Significant improvement in computational efficiency between proposed method and CST MWS







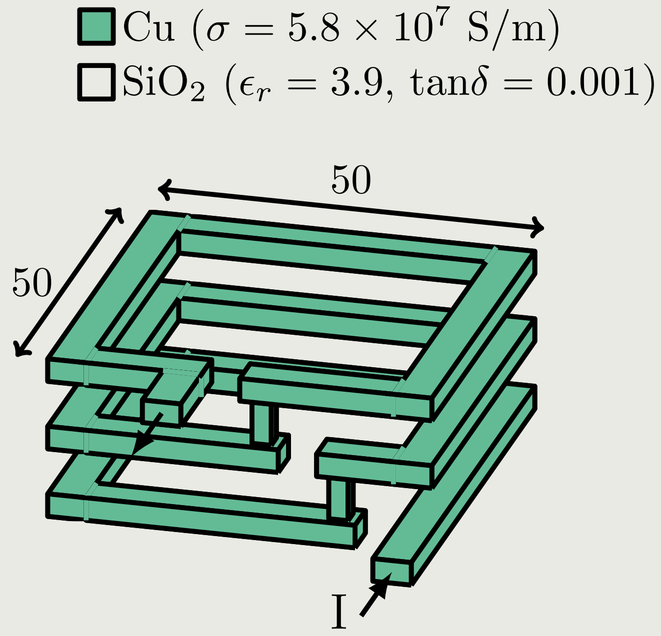
Numerical results

Stacked spiral on-chip inductor

Key component for various applications in radio frequency and mixed-signal integrated circuits

Material choices reflect typical on-chip integration practices

Unit current is injected into structure, allowing to determine the **impedance response**



Numerical results

Stacked spiral on-chip inductor

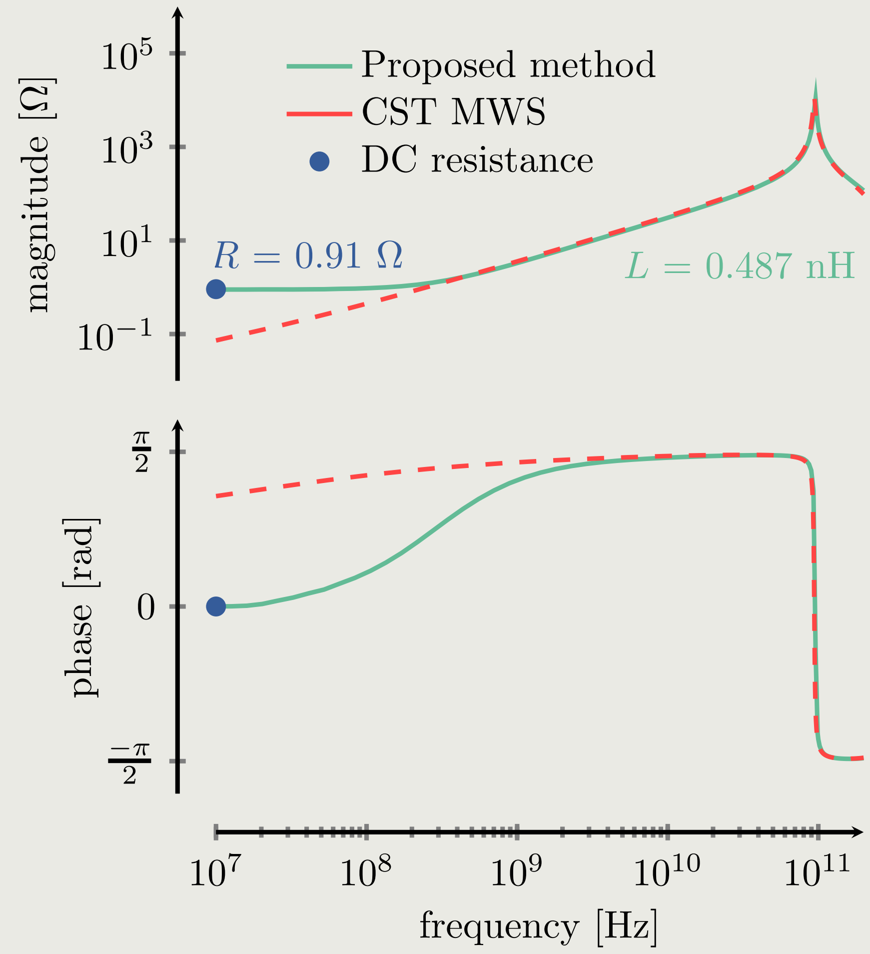
Validation with:

1. CST MWS
2. DC resistance (static solver)

Excellent agreement with CST MWS in inductive and self-resonant region

Converges to DC resistance at lower frequencies

CST MWS was unable to accurately predict the DC resistance



Conclusions

Increasing circuit speed requires accurate and efficient full-wave modeling

Proposed novel unified transform method for Maxwell’s equations in 3-D to determine DtN operators

DSA operator was constructed using these DtN operators and subsequently combined with the aEFIE

Demonstrated accurate broadband modeling of a wide range of materials using this new technique

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