

# COMPARISON OF TWO NOVEL INTEGRAL EQUATION APPROACHES FOR LOSSY CONDUCTOR MODELING

[Martijn Huynen](#) / Michiel Gossye / Daniël De Zutter / Hendrik Rogier / Dries Vande Ginste

# OUTLINE

- Motivation
- Calderón preconditioned HDC method
- 3-D differential surface admittance operator
- Examples
- Conclusions



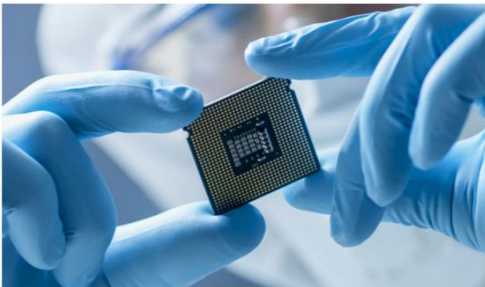
# OUTLINE

- Motivation
- Calderón preconditioned HDC method
- 3-D differential surface admittance operator
- Examples
- Conclusions



# MOTIVATION

## Industries



Semiconductors



5G to IoT

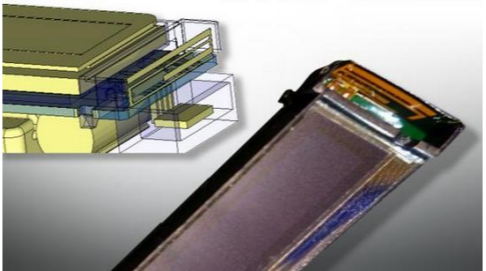


Automotive

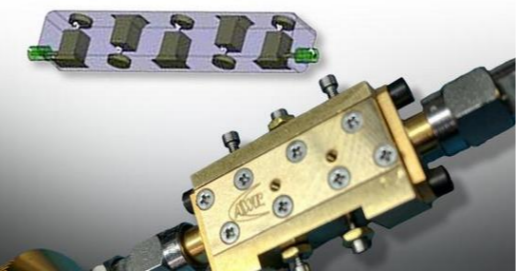


Aerospace and Defense

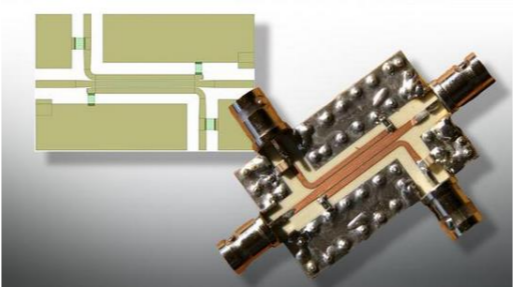
## Devices & Components



Antennas

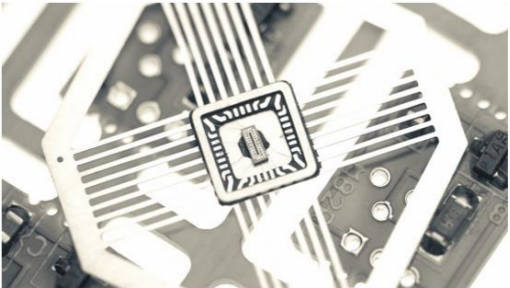


Filters

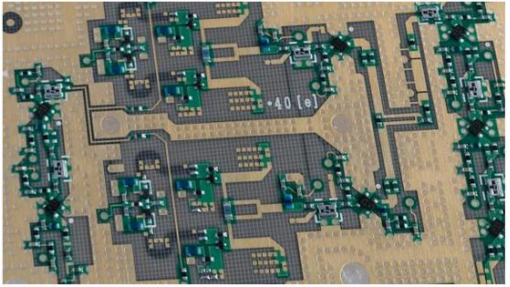


Passive components

## Technologies



MMICs



RF PCBs



Next generation ICs



# MOTIVATION

Industries

Devices & Components

Technologies

Smarter

Smaller

Faster

Increasing need for accurate simulations



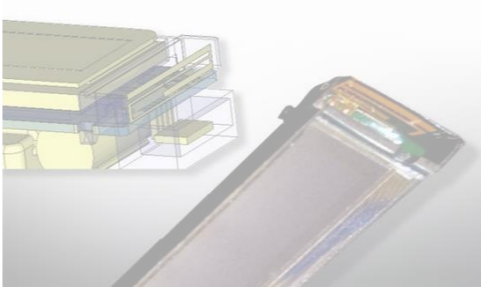
5G to IoT



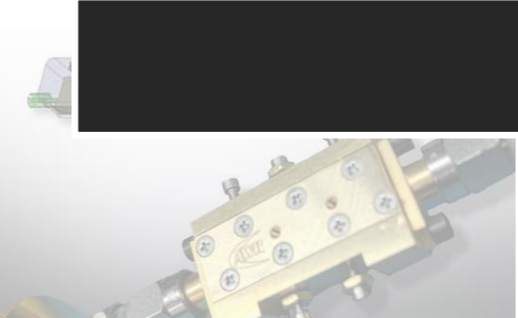
Aerospace and Defense



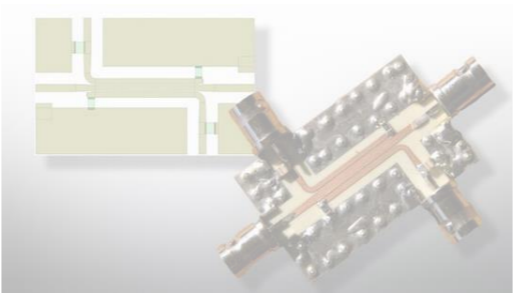
Semiconductors



Antennas



Filters



Passive components



Next generation ICs



# BOUNDARY INTEGRAL EQUATIONS

## Calderón preconditioned PMCHWT Method

Only surface discretization	Dense system matrix
Automatic inclusion radiation condition	Low-frequency & dense-mesh breakdown
Scalable through MLFMM	Difficulty numerical computation for lossy conductors

But for materials with a high dielectric contrast (HDC), e.g. good conductors, conditioning problems return

## Calderón preconditioned PMCHWT Method for HDC materials

Only surface discretization	Dense system matrix
Automatic inclusion radiation condition	Low-frequency & dense-mesh breakdown
Scalable through MLFMM	Difficulty numerical computation for lossy conductors



# THIS WORK

- Two single-source boundary integral equation (BIE) solutions that both focus on one crucial aspect:
  - Method 1: A Calderón preconditioned (CP) BIE that does not break down in the presence of HDC materials
  - Method 2: A 3-D differential surface admittance (DSA) operator that tackles the difficulty of numerically integrating the Green's function inside highly conductive dielectrics.



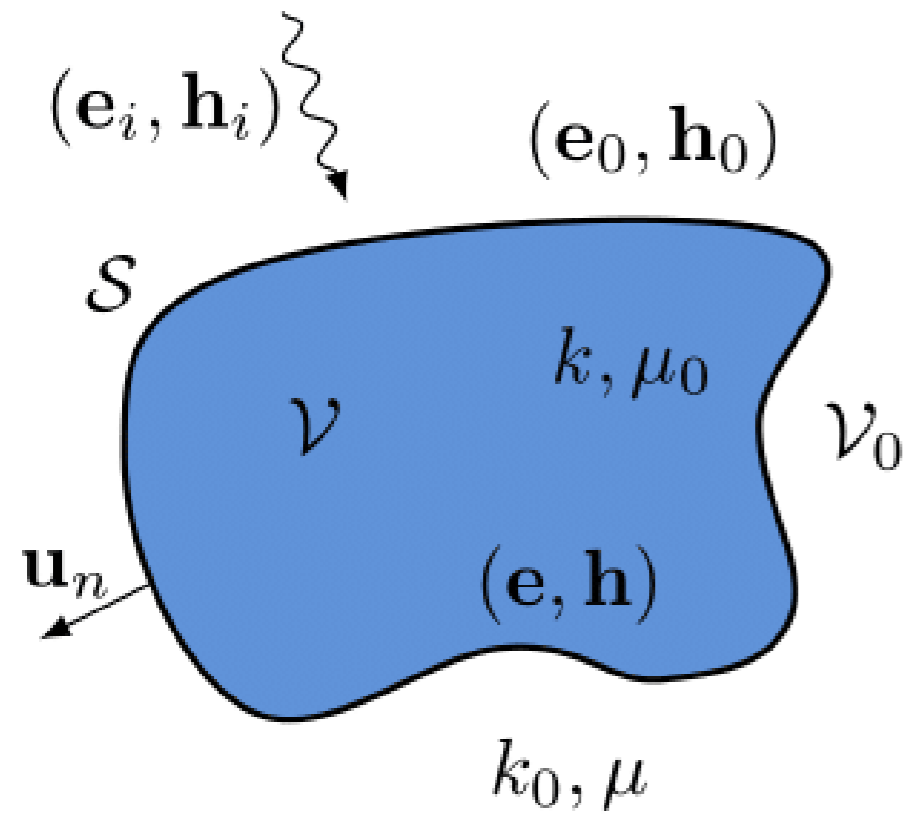
# OUTLINE

- Motivation
- **Calderón preconditioned HDC method**
- 3-D differential surface admittance operator
- Examples
- Conclusions

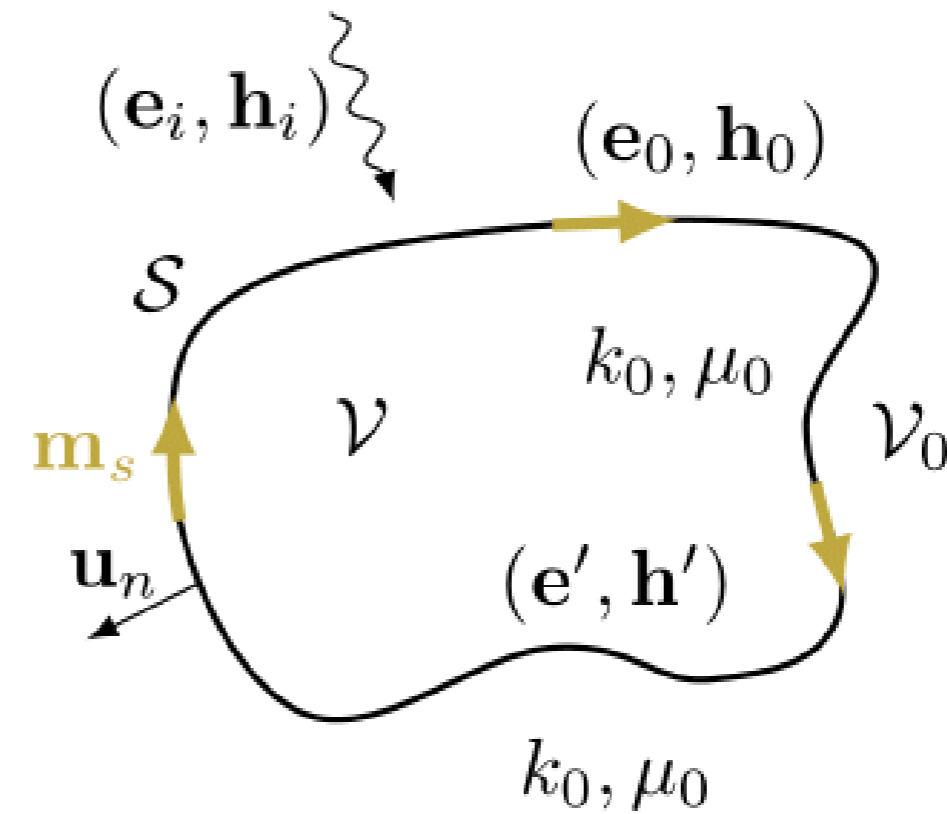




# CALDERÓN PRECONDITIONER FOR HDC MEDIA



Field equivalence



On the boundary  $S$ :

$$\left(\mathcal{K}_0 + \frac{1}{2}\right)\mathbf{m}_s + \mathbf{u}_n \times \mathbf{e} = \mathbf{u}_n \times \mathbf{e}_i$$

$$\frac{1}{\eta_0}\mathcal{T}_0\mathbf{m}_s - \mathbf{u}_n \times \mathbf{h} = -\mathbf{u}_n \times \mathbf{h}_i$$

$\mathcal{K}_0$  = Magnetic field integral operator in  $\mathcal{V}_0$

$\mathcal{T}_0$  = Electric field integral operator in  $\mathcal{V}_0$

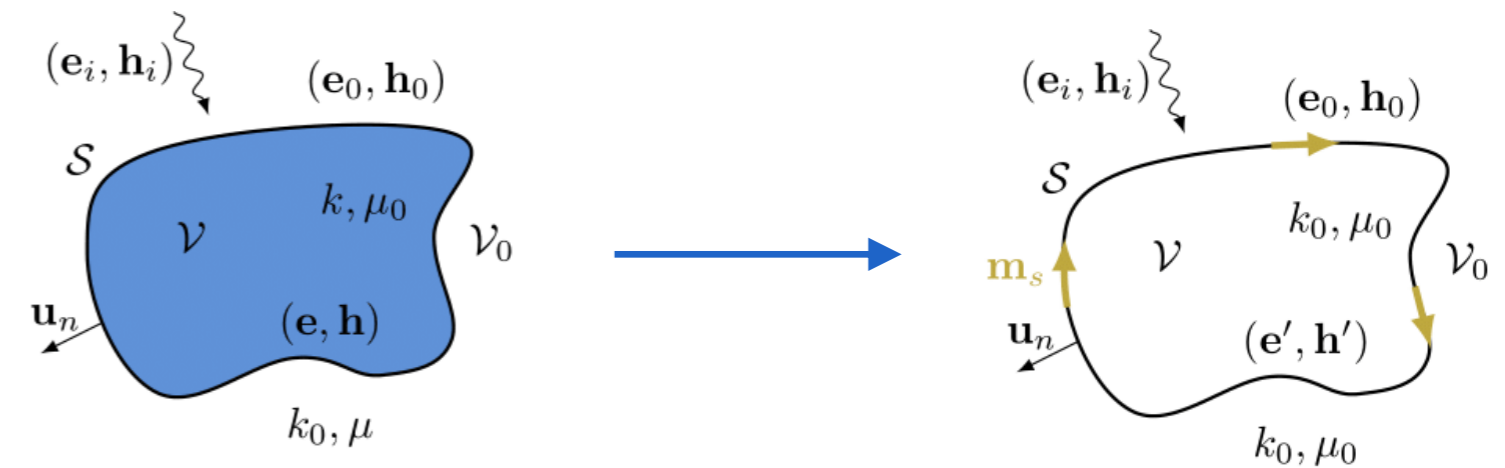
$\eta_0$  = Impedance of background medium



# CALDERÓN PRECONDITIONER FOR HDC MEDIA

Introduction of the Poincaré-Steklov operator, results in a solvable system by eliminating  $\mathbf{u}_n \times \mathbf{h}$  :

$$\mathcal{P}(\mathbf{u}_n \times \mathbf{e}) = \mathbf{u}_n \times \mathbf{h}$$



$$\left(\mathcal{K}_0 + \frac{1}{2}\right)\mathbf{m}_s + \mathbf{u}_n \times \mathbf{e} = \mathbf{u}_n \times \mathbf{e}_i$$

$$\frac{1}{\eta_0}\mathcal{T}_0\mathbf{m}_s - \mathcal{P}(\mathbf{u}_n \times \mathbf{e}) = -\mathbf{u}_n \times \mathbf{h}_i$$

$$\mathcal{P} = -\frac{1}{\eta}\mathcal{T}^{-1}\left(\mathcal{K} + \frac{1}{2}\right)$$

$\mathcal{K}$  = Magnetic field integral operator in  $\mathcal{V}$

$\mathcal{T}$  = Electric field integral operator in  $\mathcal{V}$

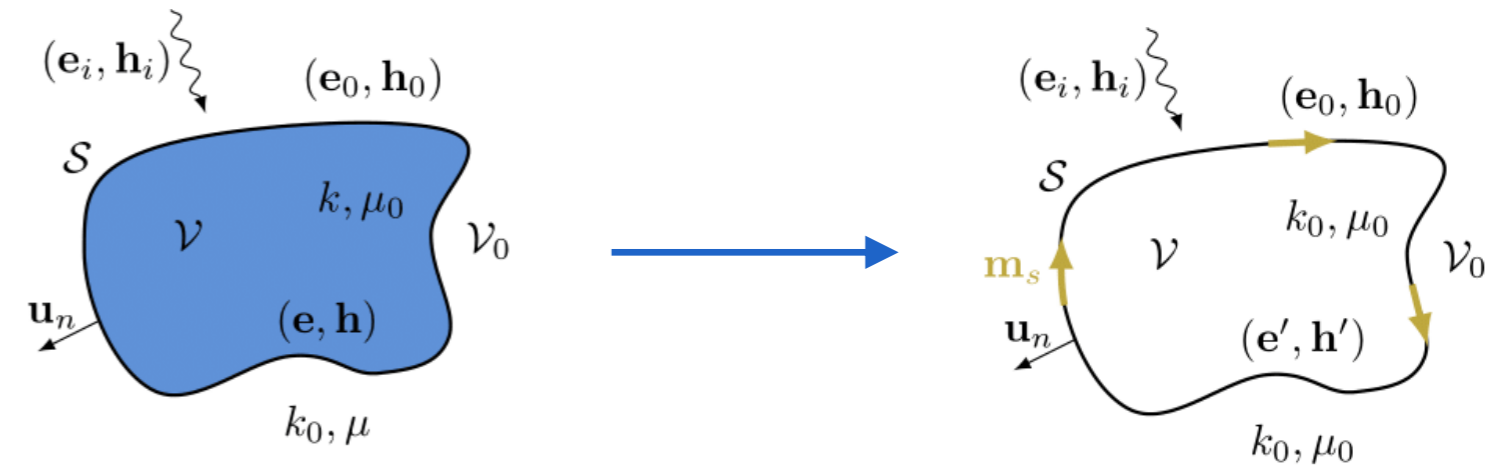
$\eta$  = Impedance of original medium

$\mathcal{P}$  = Poincaré-Steklov operator of the original medium



# CALDERÓN PRECONDITIONER FOR HDC MEDIA

$$\begin{pmatrix} \mathcal{K}_0 + \frac{1}{2} & -1 \\ \frac{\mathcal{T}_0}{\eta_0} & \mathcal{P} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{m}_s \\ -\mathbf{u}_n \times \mathbf{e} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_n \times \mathbf{e}_i \\ -\mathbf{u}_n \times \mathbf{h}_i \end{pmatrix}$$



Introduction Calderón preconditioner (CP):

- Eliminates ill-conditioning
- Avoids calculation  $\mathcal{T}^{-1}$  in Poincaré-Steklov operator

$$\begin{pmatrix} 1 & 0 \\ 0 & -\eta\mathcal{T} \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{K}_0 + \frac{1}{2} & -1 \\ -\frac{\eta}{\eta_0} \mathcal{T} \mathcal{T}_0 & \mathcal{K} + \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{m}_s \\ -\mathbf{u}_n \times \mathbf{e} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_n \times \mathbf{e}_i \\ \eta\mathcal{T} (\mathbf{u}_n \times \mathbf{h}_i) \end{pmatrix}$$

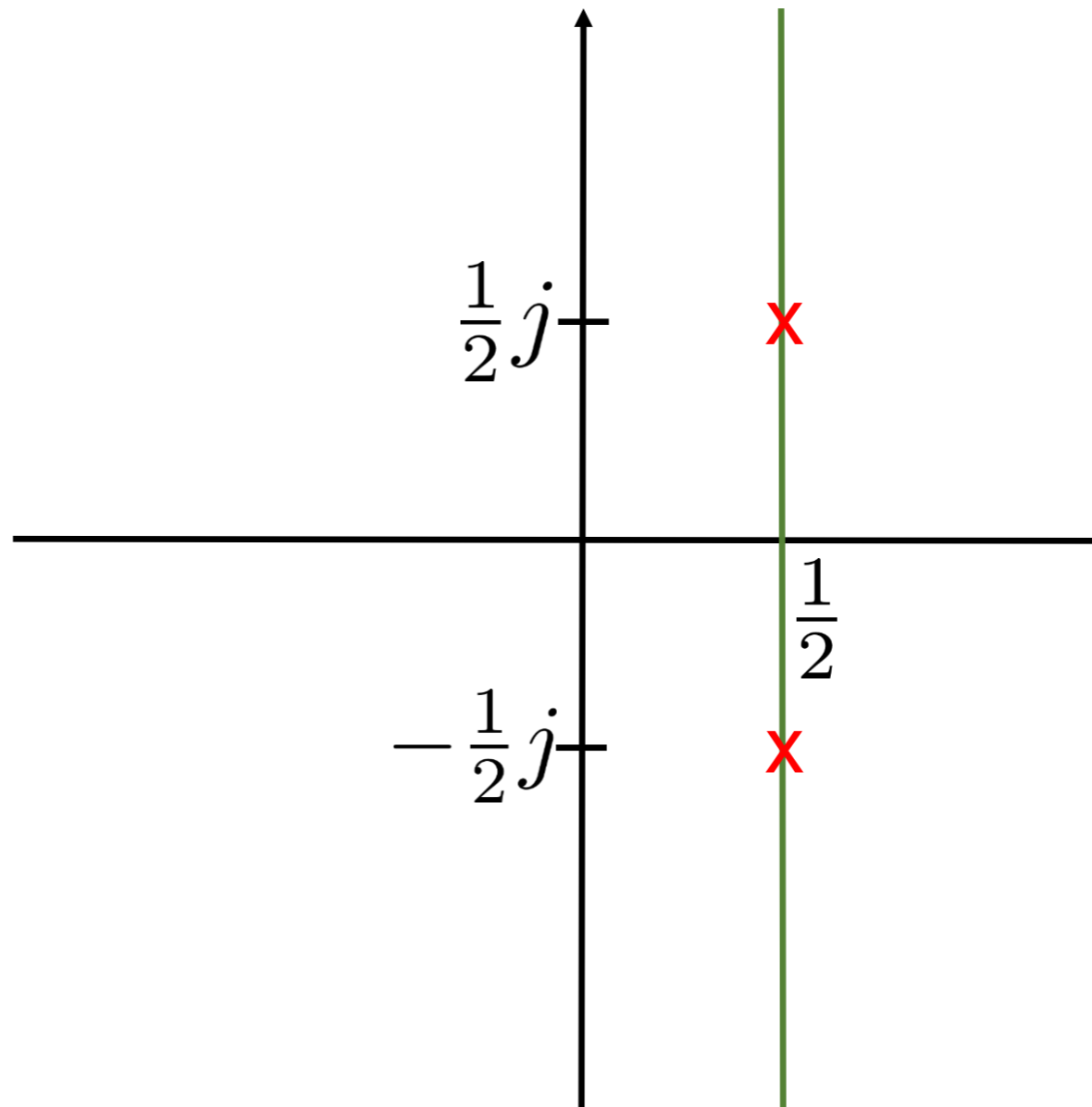


# SPECTRAL PROPERTIES

Eigenvalue accumulation points:

$$\lambda = \frac{1}{2} \pm \frac{1}{2} \sqrt{\frac{\epsilon_0}{\epsilon}} j$$

$$\lambda = \frac{1}{2} \pm \frac{1}{2} \sqrt{\frac{\mu}{\mu_0}} j$$



$\mathcal{V}_0 = \text{vacuum: } \epsilon_{r,0} = 1$   
 $\mathcal{V} = \text{vacuum: } \epsilon_r = 1$

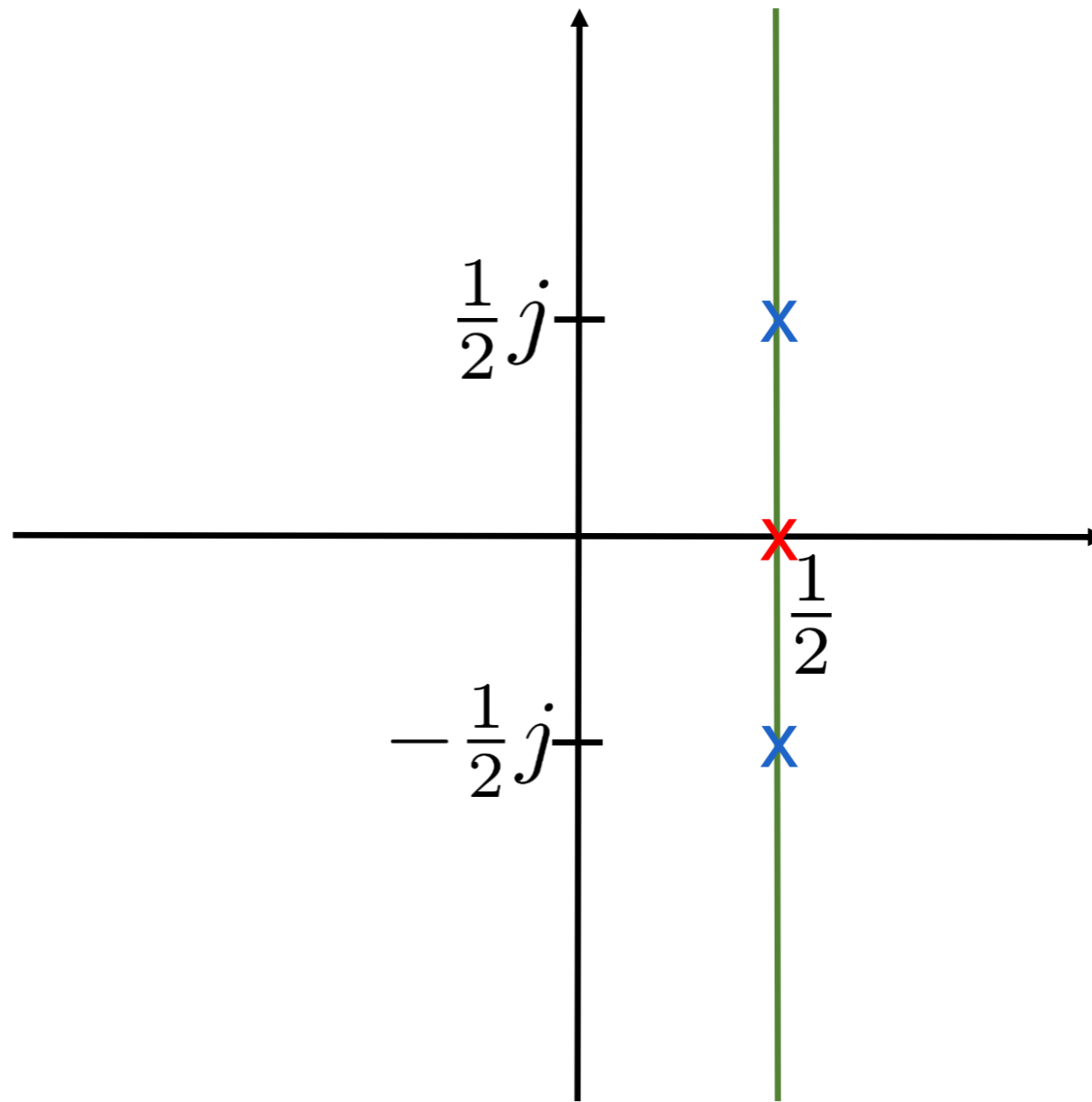


# SPECTRAL PROPERTIES

Eigenvalue accumulation points:

$$\lambda = \frac{1}{2} \pm \frac{1}{2} \sqrt{\frac{\epsilon_0}{\epsilon}} j$$

$$\lambda = \frac{1}{2} \pm \frac{1}{2} \sqrt{\frac{\mu}{\mu_0}} j$$



$\mathcal{V}_0 = \text{vacuum: } \epsilon_{r,0} = 1$   
 $\mathcal{V} = \text{copper: } |\epsilon_r| \gg 1$

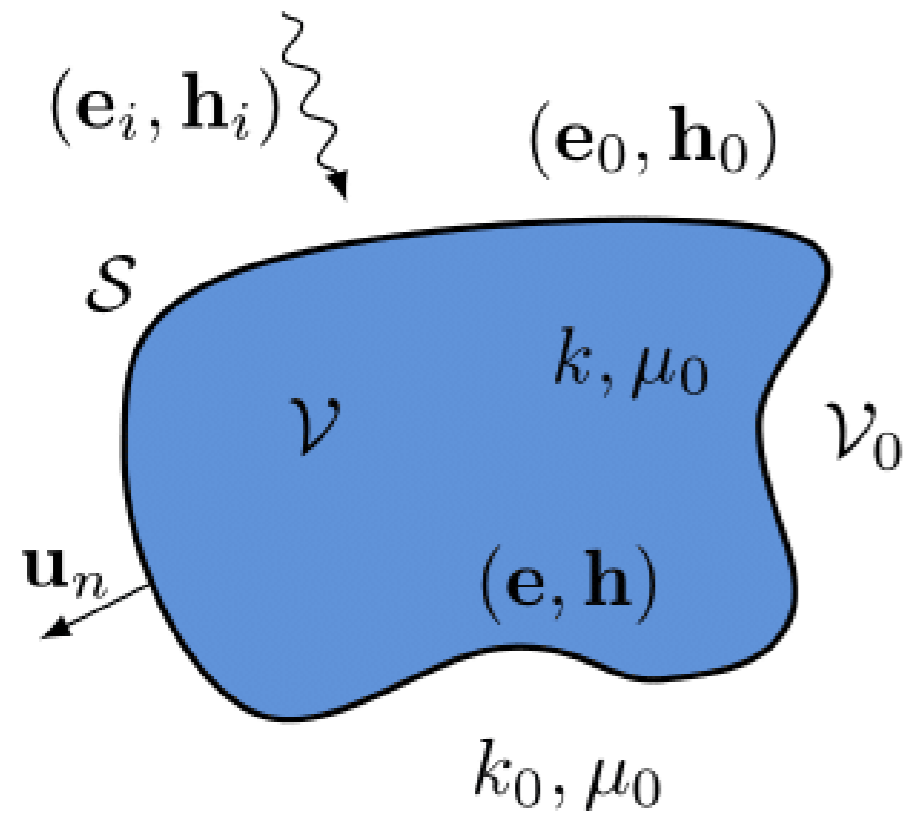


# OUTLINE

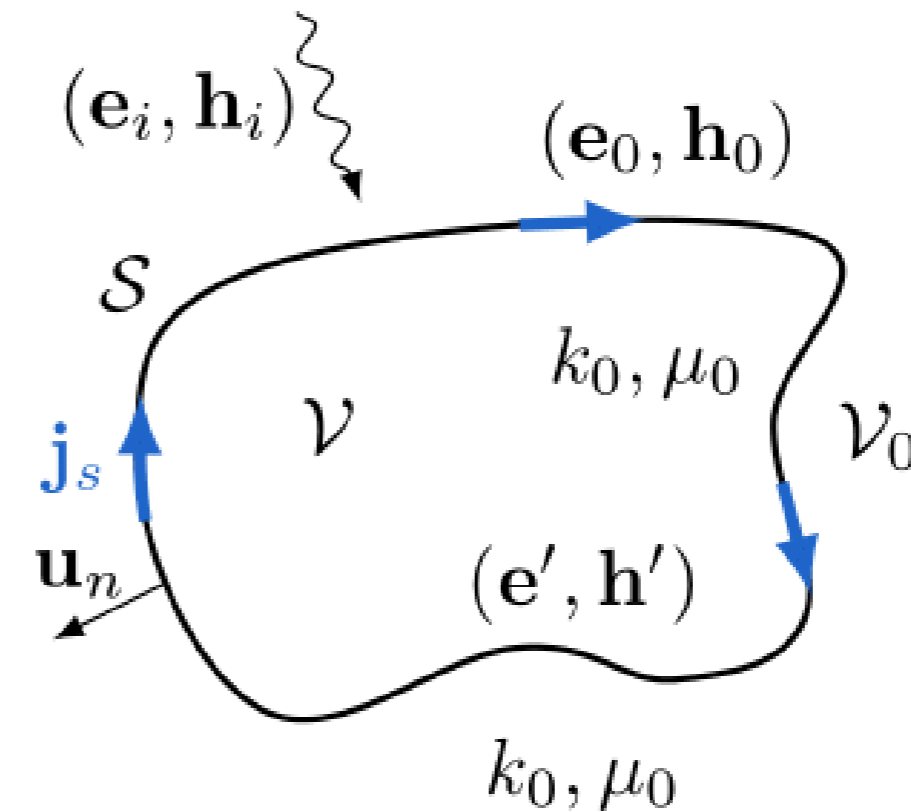
- Motivation
- Calderón preconditioned HDC method
- **3-D differential surface admittance operator**
- Examples
- Conclusions



# 3-D DIFFERENTIAL SURFACE ADMITTANCE OPERATOR



Field equivalence



On the boundary  $S$ :

$$\mathcal{P}(\mathbf{u}_n \times \mathbf{e}) = \mathbf{u}_n \times \mathbf{h}$$

$\mathcal{P}$  = Poincaré-Steklov operator of  
the original medium

On the boundary  $S$ :

$$\mathcal{P}_0(\mathbf{u}_n \times \mathbf{e}') = \mathbf{u}_n \times \mathbf{h}'$$

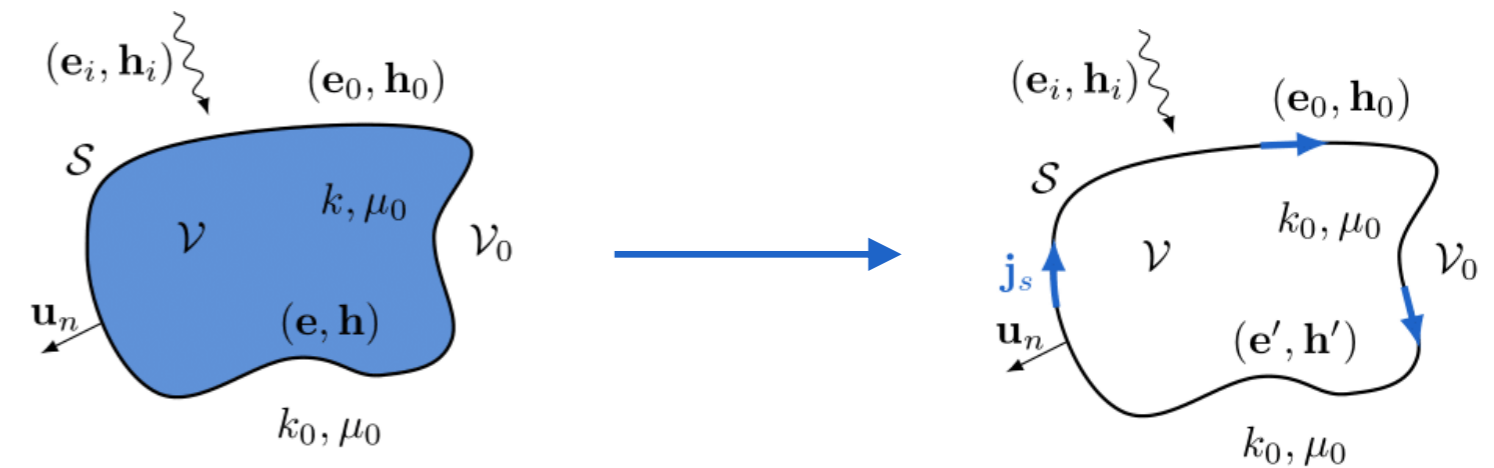
$\mathcal{P}_0$  = Poincaré-Steklov operator of  
the background medium



# 3-D DIFFERENTIAL SURFACE ADMITTANCE OPERATOR

Through the boundary conditions in both situations, both PS operators can be combined to:

$$\mathbf{j}_s = (\mathcal{P} - \mathcal{P}_0) (\mathbf{u}_n \times \mathbf{e}) = \mathcal{Y} (\mathbf{u}_n \times \mathbf{e})$$



The electric field integral equation (EFIE) completes the system of equations

$$\eta_0 \mathcal{T} \mathbf{j}_s + \mathbf{u}_n \times \mathbf{e} = \mathbf{u}_n \times \mathbf{e}_i$$

$\mathcal{Y}$  could be computed directly, but the computation of  $\mathcal{P}$  involves bothersome numerical integrals  $\otimes$

Novel alternative method that avoids these integrals...  $\checkmark$

$\mathcal{Y} =$  Differential surface admittance operator





# 3-D DIFFERENTIAL SURFACE ADMITTANCE OPERATOR

$$\mathcal{Y}[\mathbf{x}] = \eta \sum_{\nu} \frac{\mathcal{K}_{\nu}}{\mathcal{N}_{\nu}^2} \left[ \int_{\mathcal{S}} (\mathbf{u}_n \times \mathbf{h}_{mnp}) \cdot \mathbf{x} dS \right] (\mathbf{u}_n \times \mathbf{h}_{mnp})$$

$(k^2 - k_0^2) / j\omega\mu_0$       Normalization constant

Magnetic eigenmodes

- ✓ Valid for all materials (including good conductors) over a broad frequency range
- ✗ Requires eigenmodes of the volume  $\mathcal{V}$   
 $\Rightarrow \mathcal{V}$  should be a canonical shape
- ✓ Does not rely on the Green's function in the medium  
 $\Rightarrow$  integrals are material independent

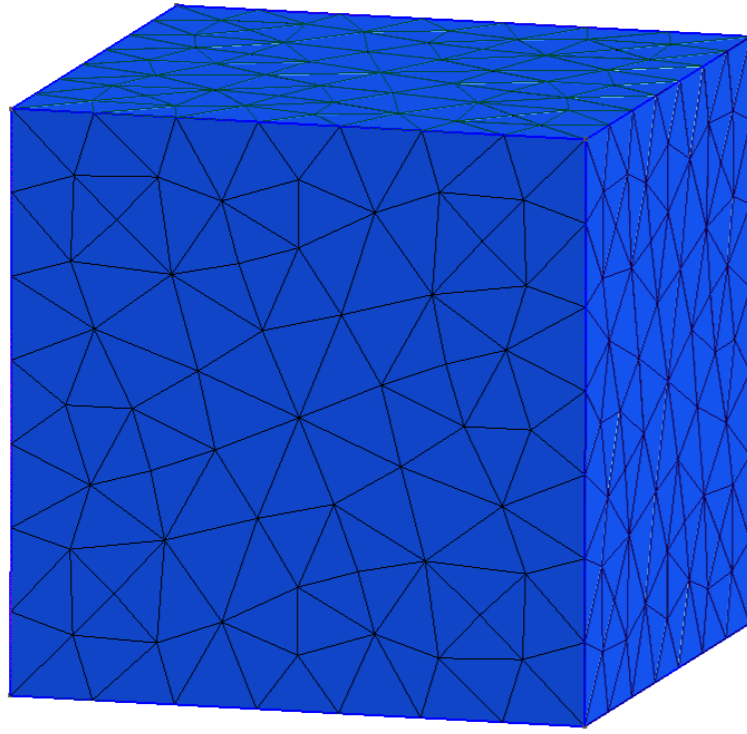


# OUTLINE

- Motivation
- Calderón preconditioned HDC method
- 3-D differential surface admittance operator
- **Examples**
- Conclusions

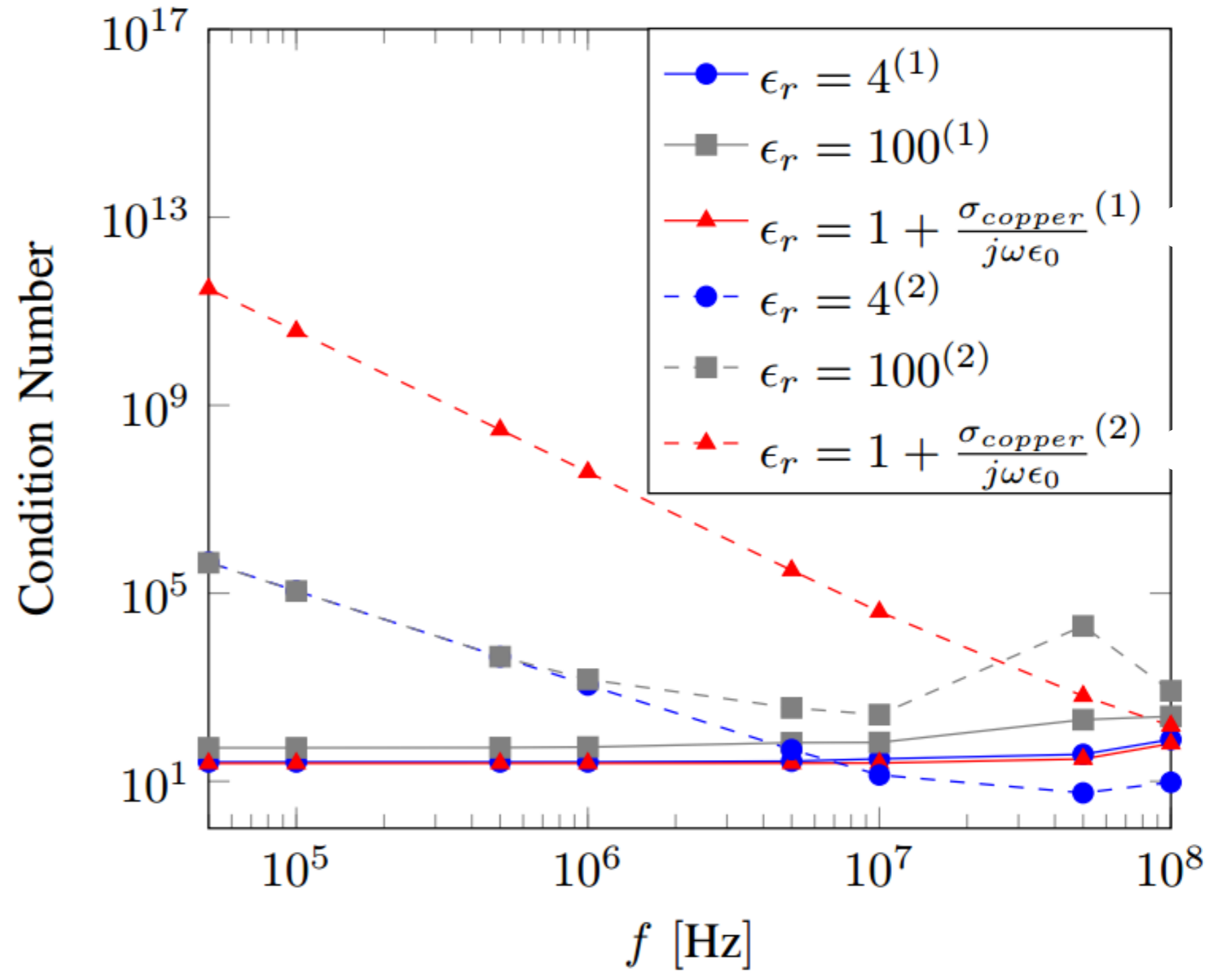


# VALIDATION METHOD 1



Side = 2 m

Low frequency breakdown does not occur:  
Condition number converges for decreasing frequency



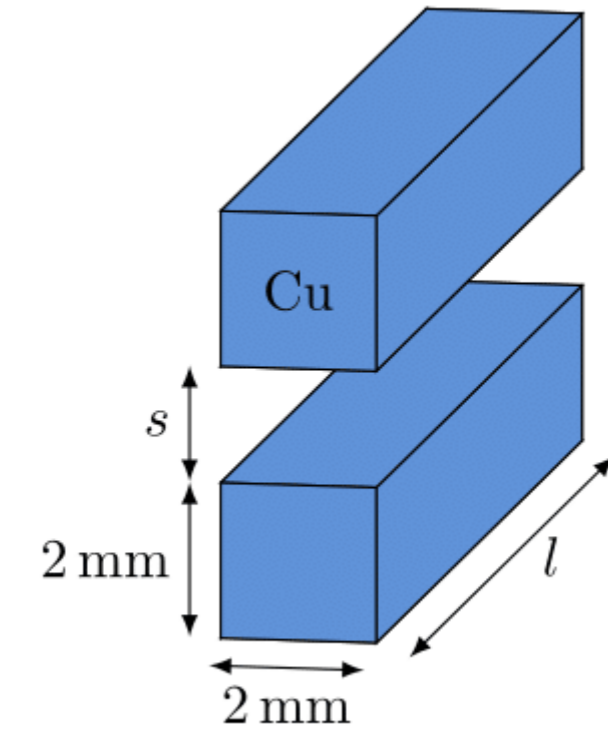
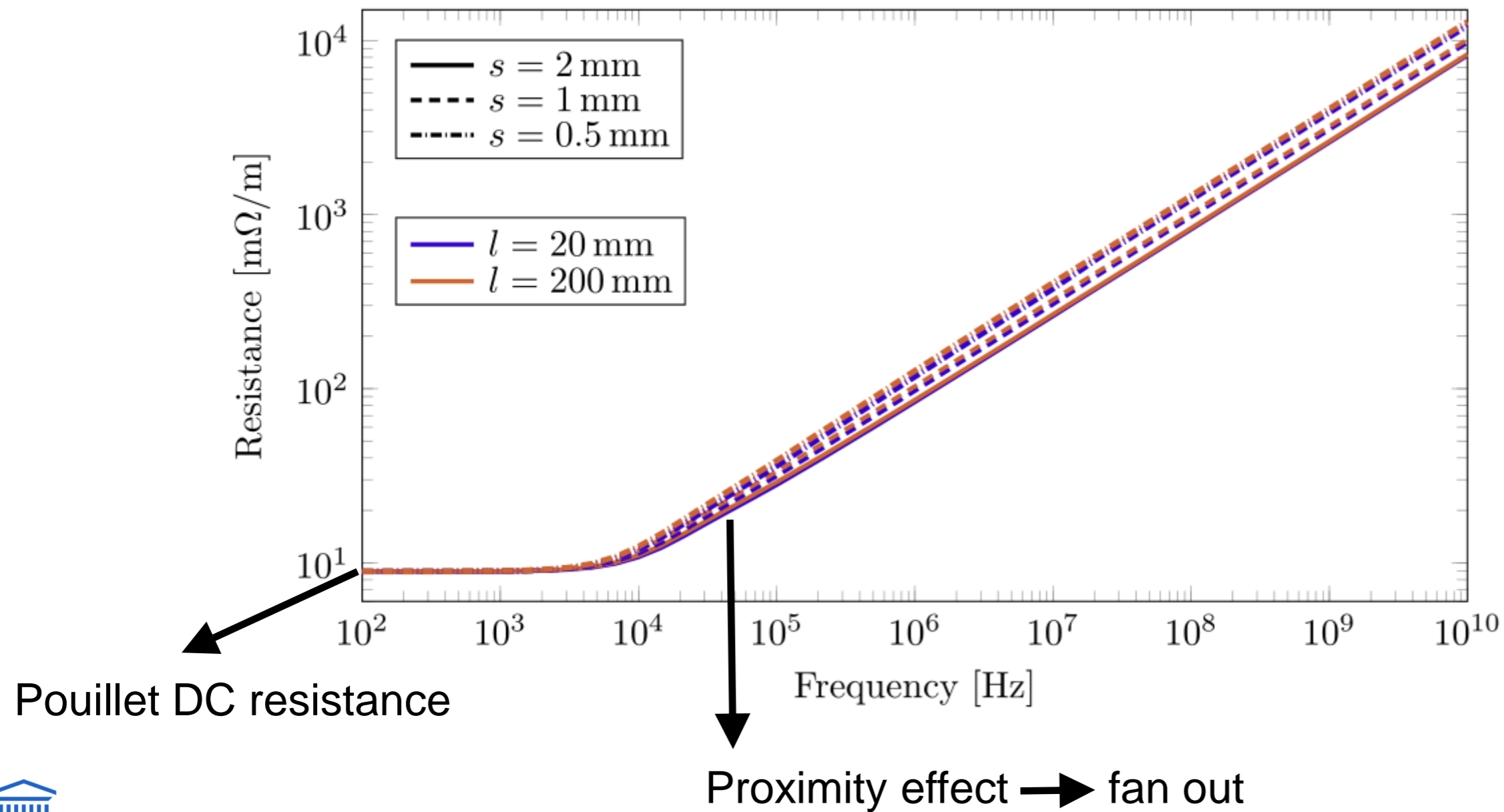
Full line = method 1

Dashed line = CP-PMCHWT



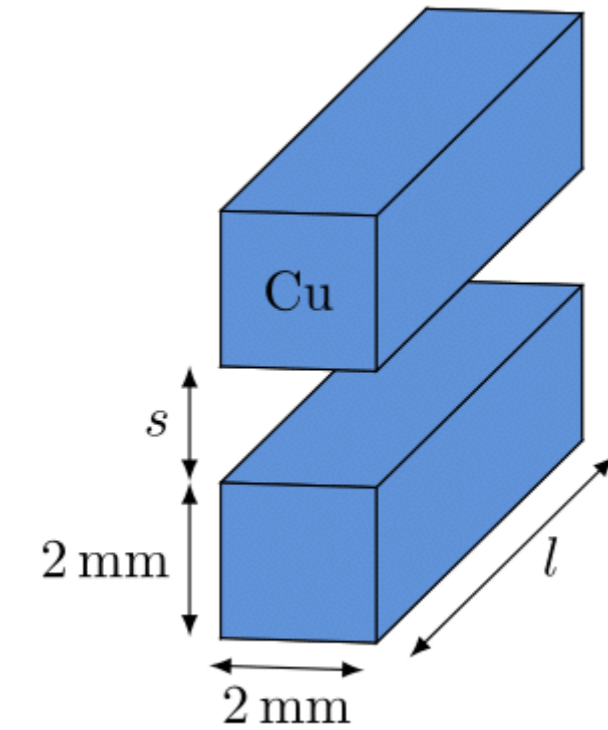
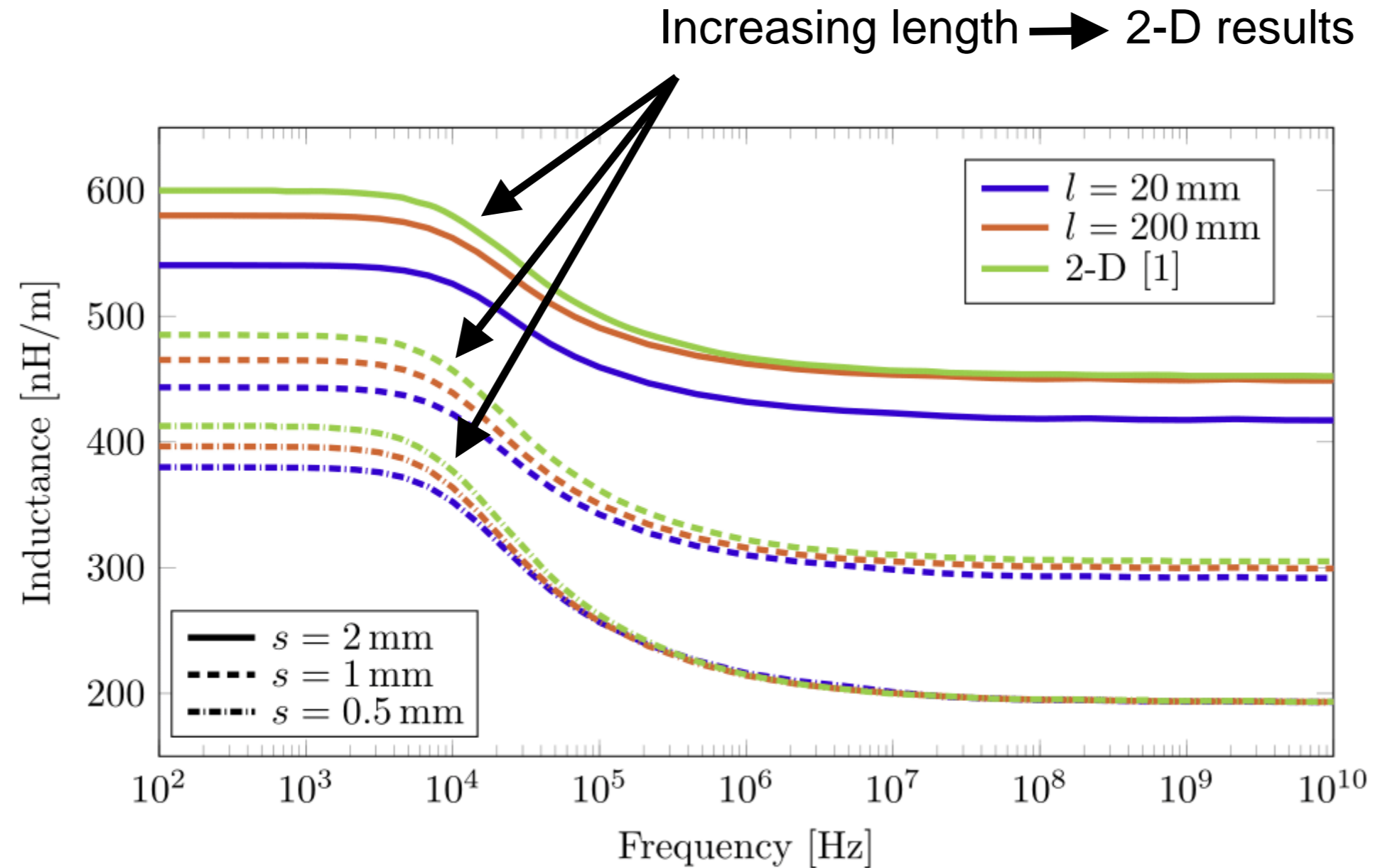
# VALIDATION METHOD 2 (1)

Normalized resistance = total resistance (3-D) / length

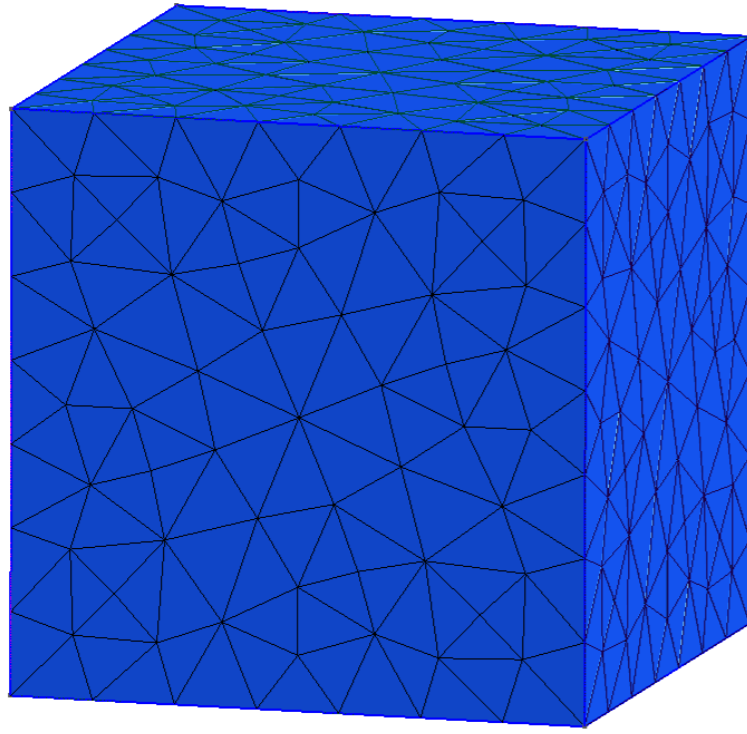


# VALIDATION METHOD 2 (2)

## Normalized inductance



# COMPARISON METHOD 1 & 2: CUBE SCATTERING

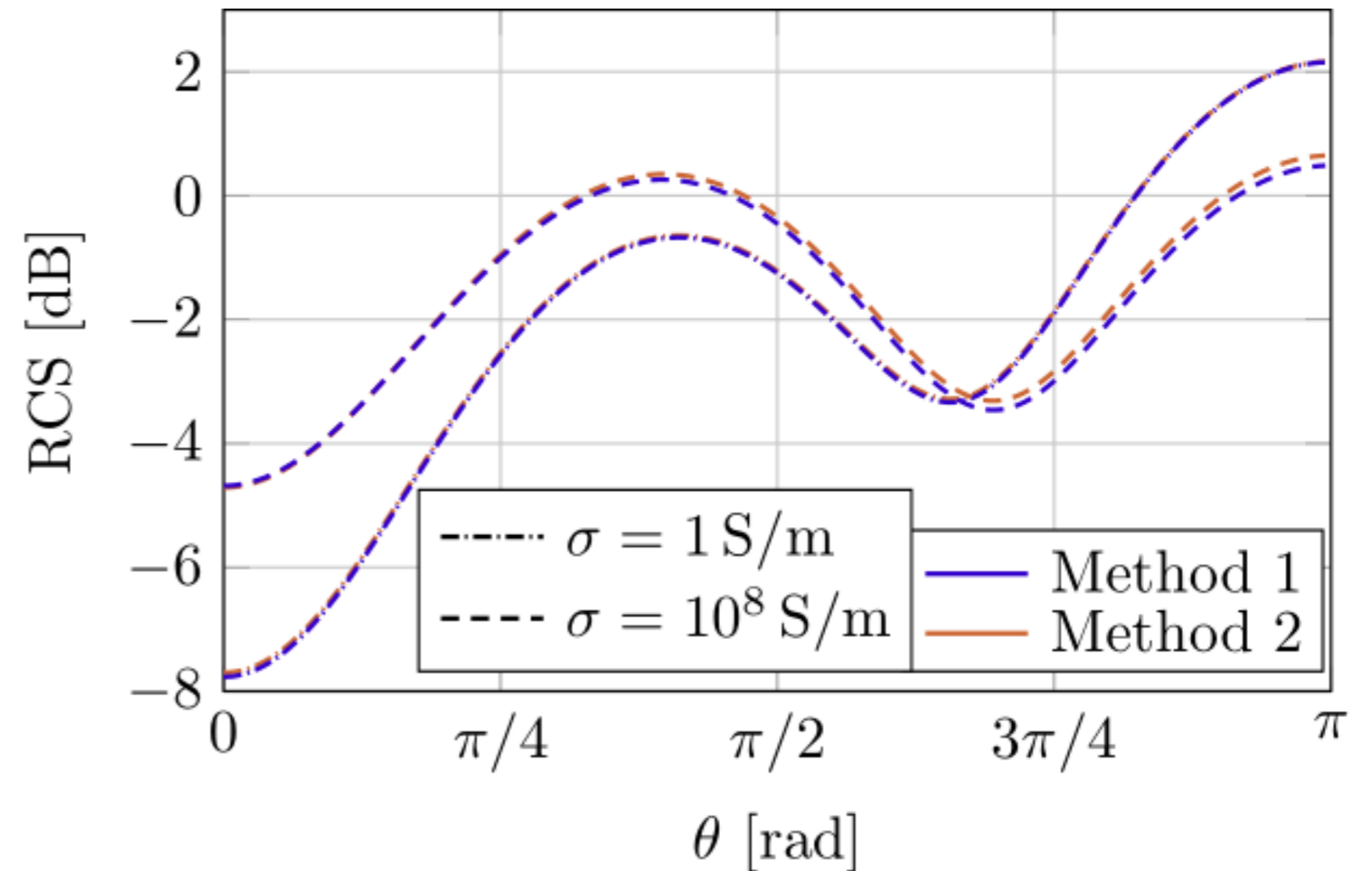


Side = 0.5 m      Frequency = 200 MHz

# mesh elements:

Method 1: 936 (generated triangular mesh)

Method 2: 432 (structured rectangular mesh)

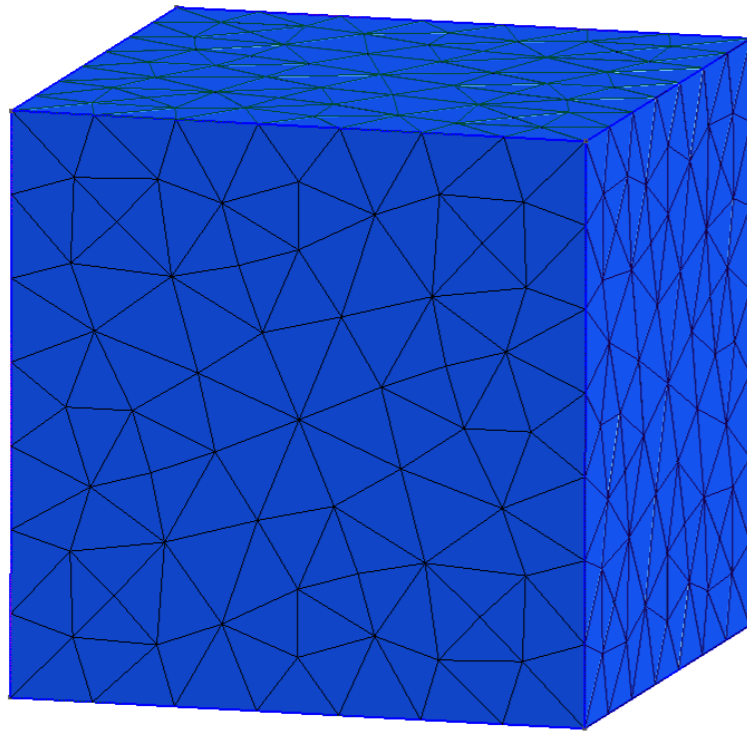


Very good agreement between both methods

Small deviation due to coarse mesh



# COMPARISON METHOD 1 & 2: CUBE SCATTERING

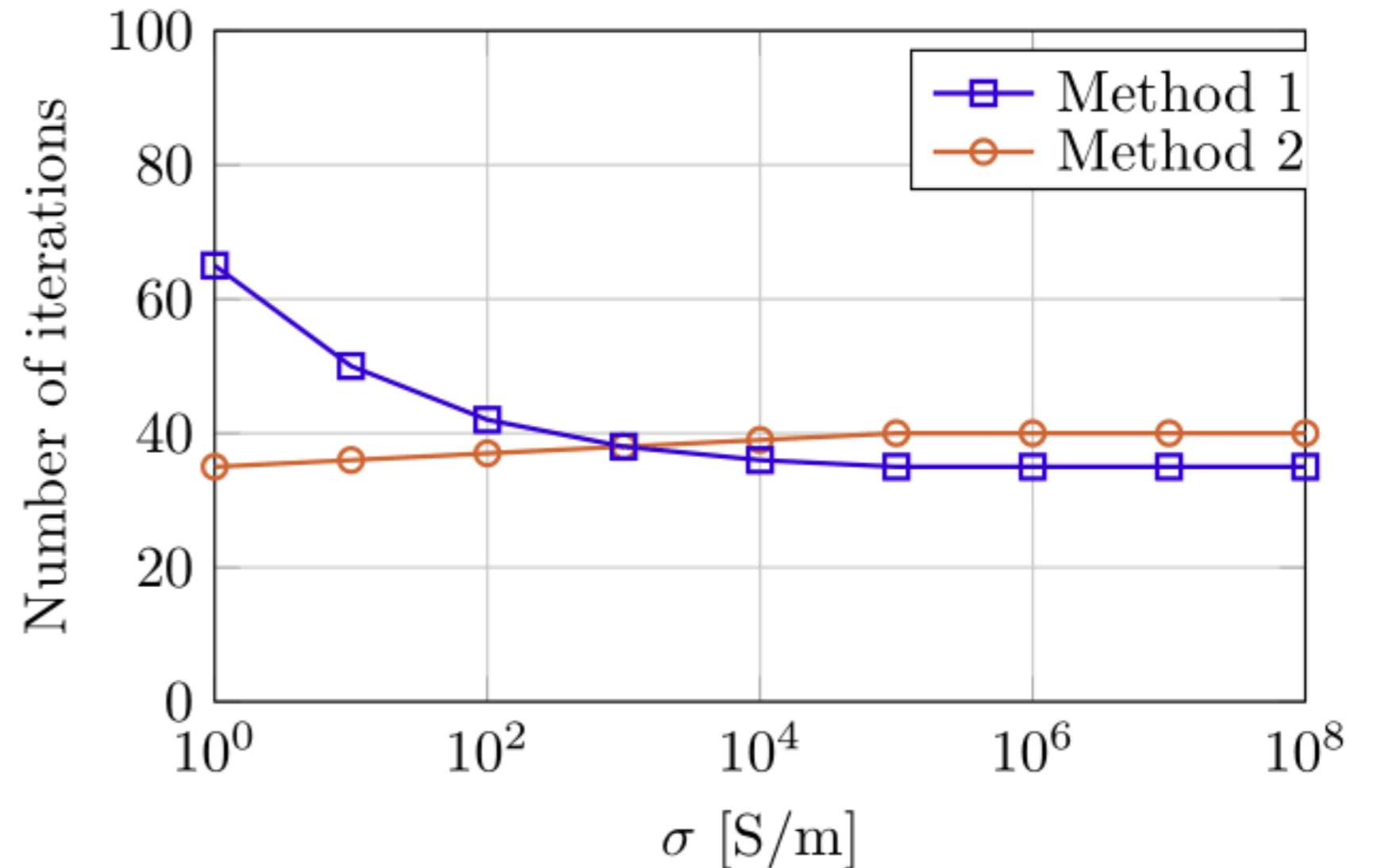


Side = 0.5 m      Frequency = 200 MHz

# mesh elements:

Method 1: 936 (generated triangular mesh)

Method 2: 432 (structured rectangular mesh)

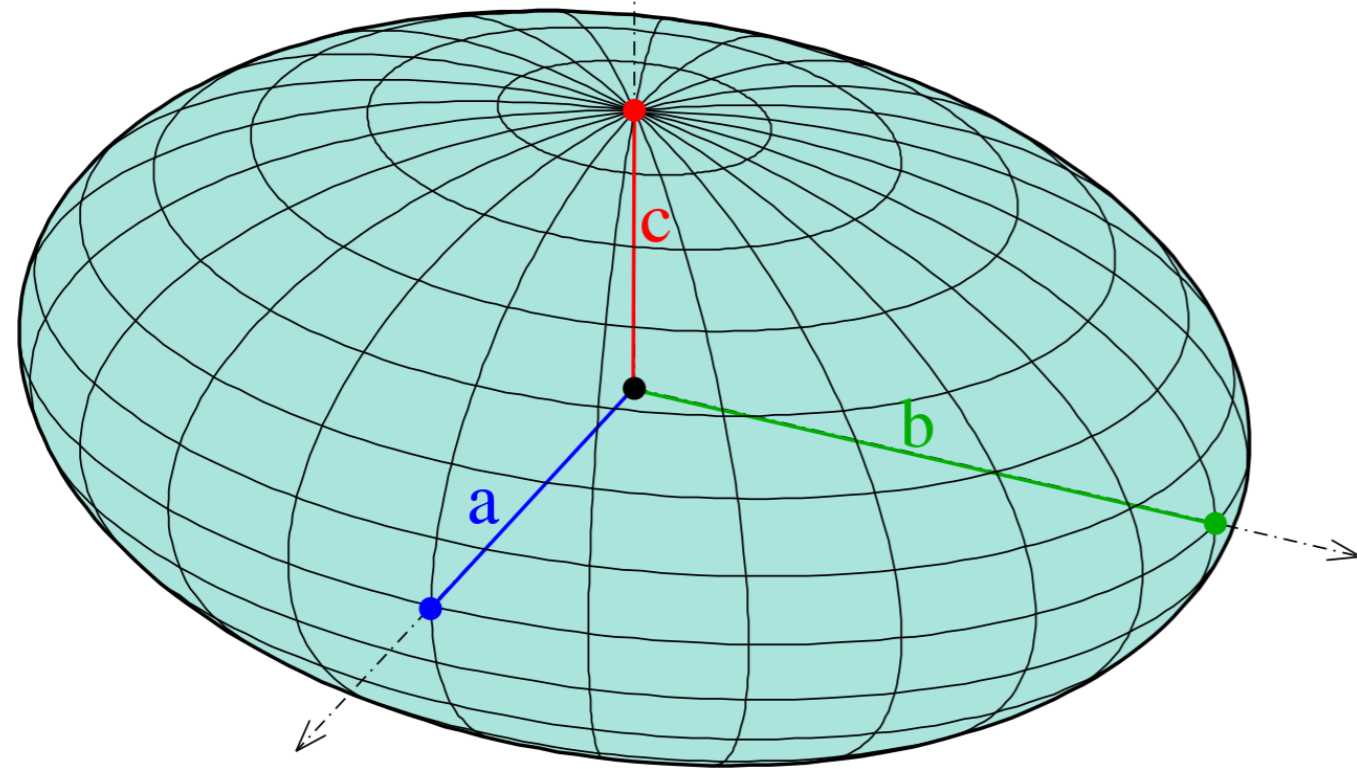


Similar # iterations for solution

Method 1 outperforms method 2 for HDC despite larger system matrix

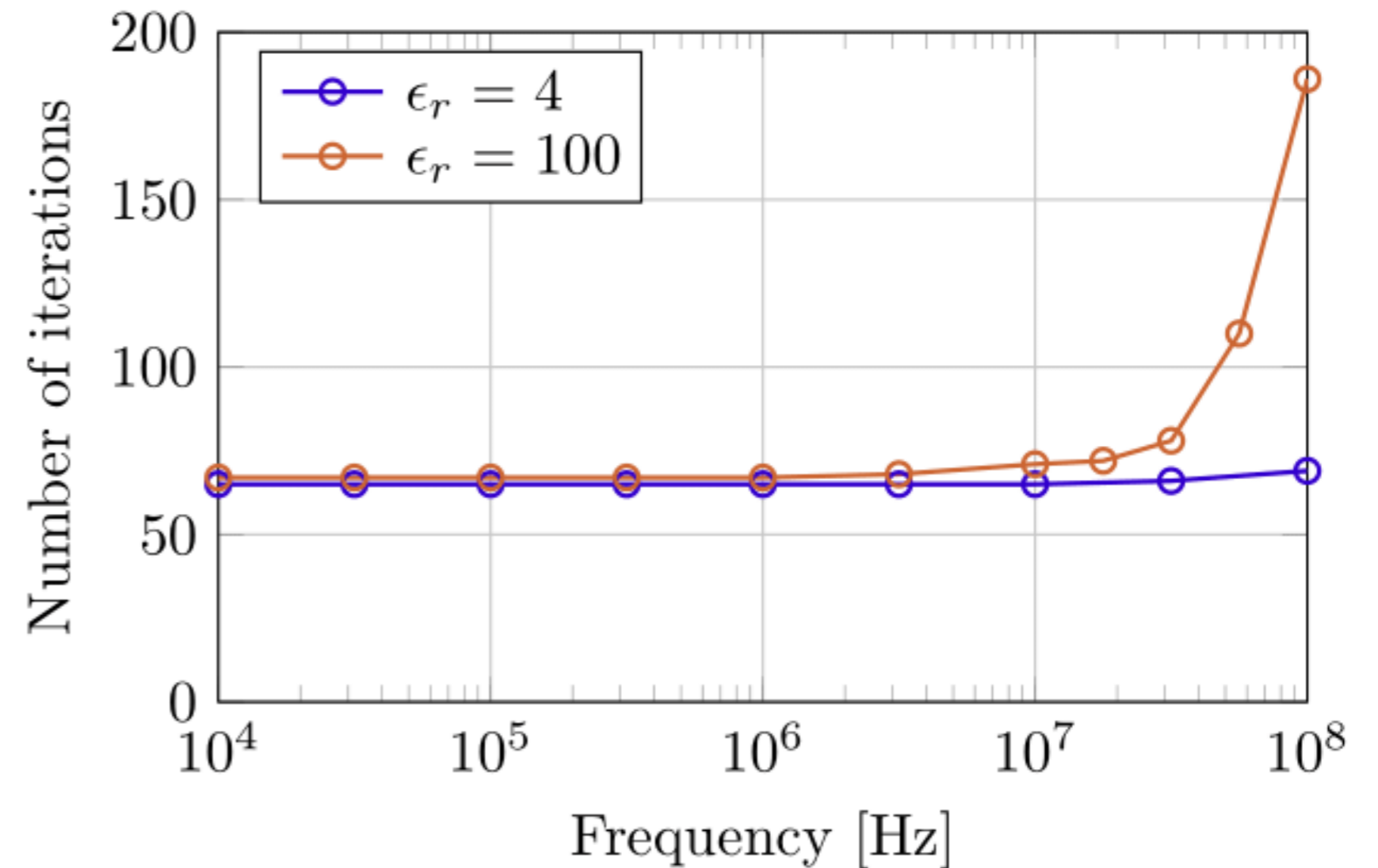


# APPLICATION METHOD 1: ELLIPSOID SCATTERING



$a = 0.6 \text{ m}$ ,  $b = 0.4 \text{ m}$ ,  $c = 0.2 \text{ m}$

Frequency = 200 MHz



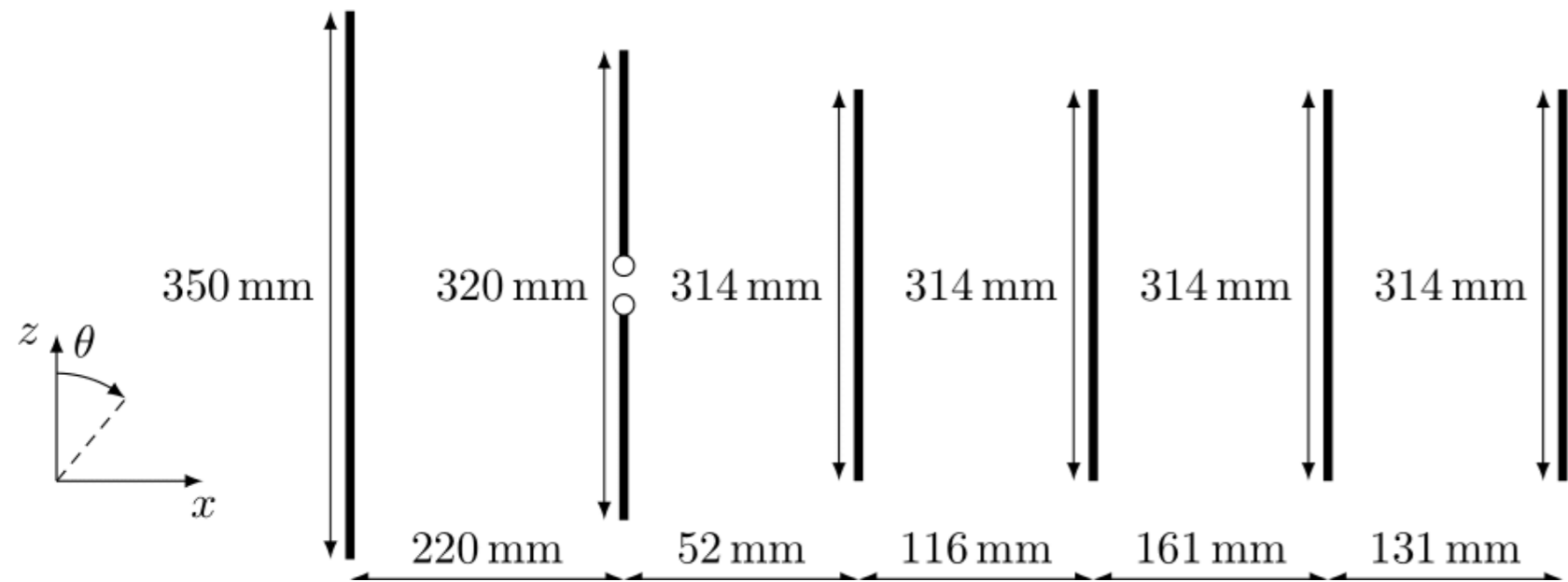
Stable number of iterations over a wide frequency range

Increase for HDC at highest frequency due to internal resonances

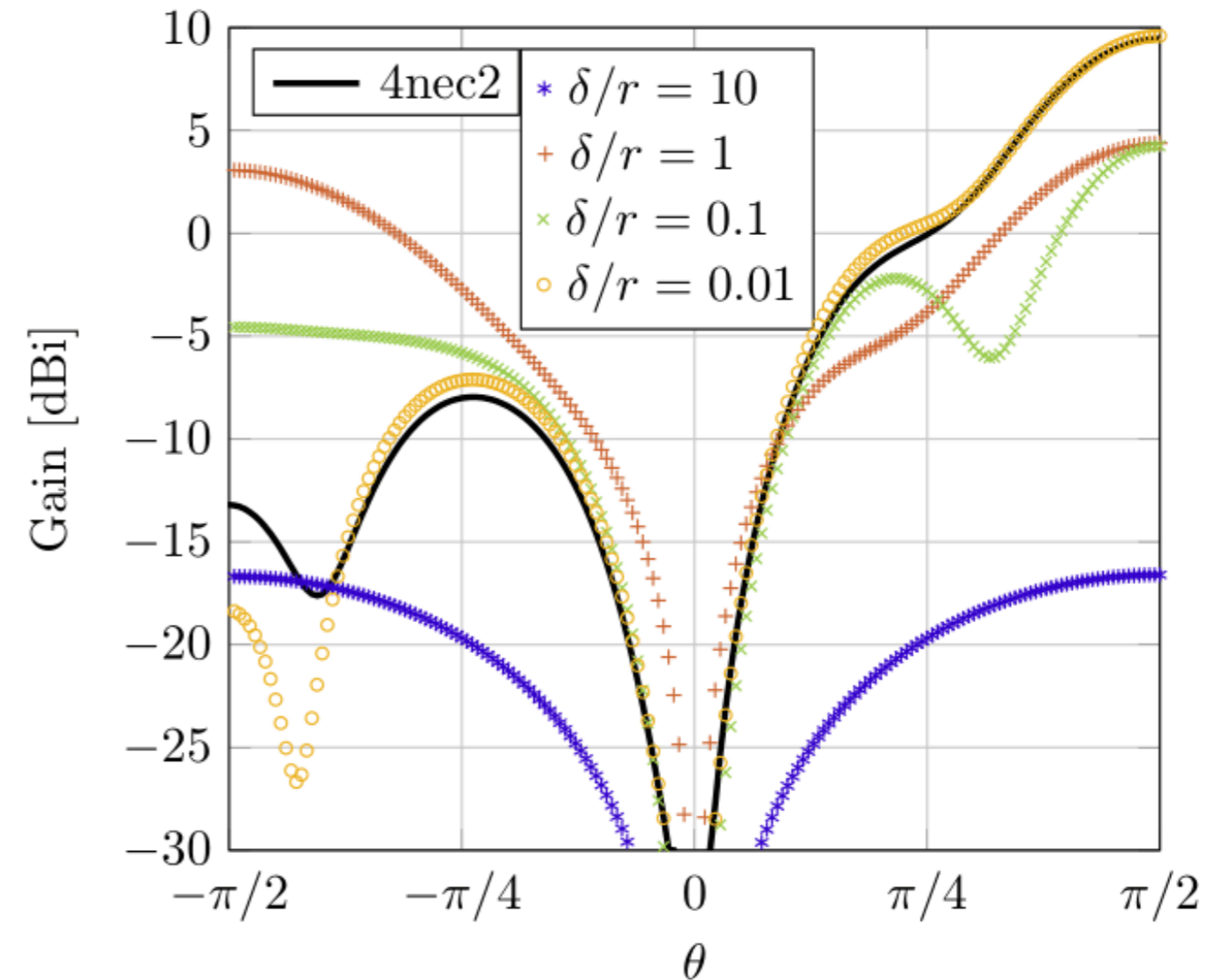




# APPLICATION METHOD 2: YAGI-UDA



Frequency = 434 MHz  
(ISM band)



For poor conductive elements, parasitic elements are transparent  $\rightarrow$  responsive resembles (inefficient) dipole

Once skin effect appears, gain increases and Yagi-Uda operation starts shaping the gain pattern



# OUTLINE

- Motivation
- Calderón preconditioned HDC method
- 3-D differential surface admittance operator
- Examples
- **Conclusions**



# CONCLUSIONS

- Two novel boundary integral equation methods
  - Single-source formulations
  - Well equipped to handle good conductors & HDC
- New Calderón preconditioner
  - No low-frequency or dense-mesh breakdown for HDC
- 3-D differential surface admittance operator
  - No cumbersome integrals involving HDC's Green's function
  - Broadband model



# Martijn Huynen

Post-doctoral researcher

QUEST LAB & ELECTROMAGNETICS GROUP

E Martijn.Huynen@ugent.be

T +32 9 331 48 81

M +32 400 00 00 00

[www.ugent.be](http://www.ugent.be)

[www.imec.be](http://www.imec.be)



Universiteit Gent



@ugent



Ghent University

