Interconnect Modeling using a Surface Admittance Operator Derived with the Fokas Method

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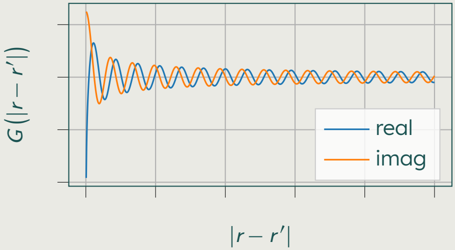
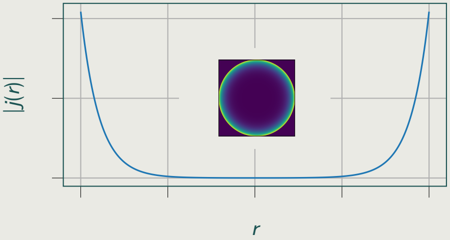
Context

- Trends in present-day electronic devices
  - Increasing operating frequencies
  - Continuing miniaturization
- Accurate **full-wave modeling tools** are required
- **Skin and proximity effects** in modern interconnects are challenging
  - Green's function integrals in surface-based methods
  - Fine discretization in volumetric methods



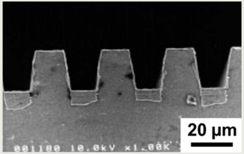


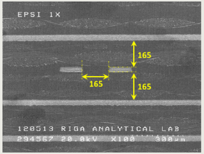
Context

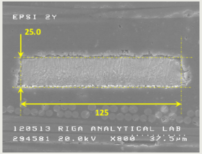
Differential surface admittance operator[1]: exact and global boundary condition

- **Current implementations**
  - Limited to canonical shapes[2]
  - *or* reintroduction of conductive medium Green's function[3]
- **Proposed:** construction invoking **Fokas method**
  $\longrightarrow$ arbitrary homogeneous material properties and **polygonal shapes**

1. D. De Zutter and L. Knockaert, "Skin effect modeling based on a differential surface admittance operator," *IEEE Trans. Microw. Theory Techn.*, 2005
2. T. Demeester and D. De Zutter, "Construction of the Dirichlet to Neumann boundary operator for triangles and applications in the analysis of polygonal conductors," *IEEE Trans. Microw. Theory Techn.*, 2010
3. U. R. Patel and P. Triverio, "Skin effect modeling in conductors of arbitrary shape through a surface admittance operator and the contour integral method," *IEEE Trans. Microw. Theory Techn.*, 2016





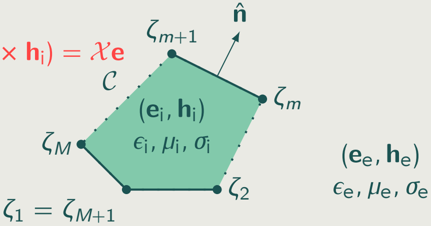


Theoretical formulation

Construction of the differential surface admittance (DSA) operator

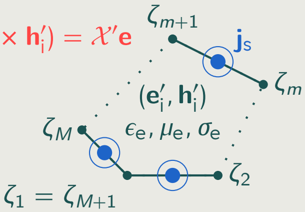
Original situation

$-\jmath\omega\mu_\mathrm{i}(\hat{\mathbf{n}} \times \mathbf{h}_\mathrm{i}) = \mathcal{X}\mathbf{e}$



Equivalent situation

$-\jmath\omega\mu_\mathrm{e}(\hat{\mathbf{n}} \times \mathbf{h}'_\mathrm{i}) = \mathcal{X}'\mathbf{e}$



$$\mathbf{j}_\mathrm{s} = \hat{\mathbf{n}} \times (\mathbf{h}_\mathrm{i} - \mathbf{h}'_\mathrm{i}) = \left(\frac{\mathcal{X}'}{\jmath\omega\mu_\mathrm{e}} - \frac{\mathcal{X}}{\jmath\omega\mu_\mathrm{i}} \right) \mathbf{e} \triangleq \mathcal{Y}\mathbf{e}$$

DSA operator $\mathcal{Y}$

Dirichlet-to-Neumann (DtN) operator $\mathcal{X}'$ mapping $\mathbf{e}$ to its normal derivative

Proposed: new approach to construct the DSA operator

Theoretical formulation

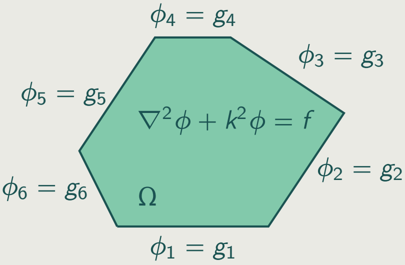
The Fokas method: general concept

Solution of **boundary value problem** (BVP) on n-sided polygon Ω

- Governed by partial differential equation (e.g., Helmholtz with real k)
- Subject to conditions on the sides $\partial\Omega_j$ (e.g., Dirichlet)
- Find unknown boundary values (e.g., Neumann)

$$\begin{cases} \nabla^2\phi + k^2\phi = f & \text{in } \Omega,\ k \in \mathbb{R} \\ \phi_j = g_j & \text{on } \partial\Omega_j,\ j \in \{1, \ldots, n\} \end{cases}$$

$$\frac{\partial \phi_j}{\partial n}?$$



A. S. Fokas, "A unified transform method for solving linear and certain nonlinear PDEs," *Proc. Royal Soc. A*, 1997

Theoretical formulation

The Fokas method: interconnect modeling

- **Novel application** of **extended** Fokas method: determine $\mathcal{X}$ and $\mathcal{X}'$
  - Relation between $\phi = e_z$ and its normal derivative $\frac{\partial \phi}{\partial n} = -\jmath \omega \mu_{\{i,e\}} h_{tan}^{\{i,e\}}$
  - e_z satisfies Helmholtz equations $\nabla^2 e_z + k_{\{i,e\}}^2 e_z = 0$ with $k_{\{i,e\}} \in \mathbb{C}$
- Reformulate well-known BVP using Fokas method's **global relation**

$$F(\lambda) = \int_{\mathcal{C}} \exp\left[-\frac{\jmath k}{2}\left(\frac{\tilde{\zeta}}{\lambda} + \lambda\zeta\right)\right] \times \left[\frac{k\phi}{2}\left(\lambda\,\mathrm{d}\zeta - \frac{\mathrm{d}\tilde{\zeta}}{\lambda}\right) + \frac{\partial\phi}{\partial n}\,\mathrm{d}c\right] = 0,\ \forall\lambda \in \mathbb{C}$$

ζ: position on $\mathcal{C}$
λ: collocation points

- Cast ϕ on basis of P Legendre polynomials and evaluate F in L well-chosen λ's

Theoretical formulation

Extraction of resistance and inductance per unit of length

- $L \gg P$: overdetermined system in terms of unknown coefficients for $\frac{\partial \phi}{\partial n}$
- Solution and projection on pulse basis yields discretized equation

$$\mathbf{j}_\mathrm{s} = \left(\frac{\mathcal{X}'}{\jmath\omega\mu_\mathrm{e}} - \frac{\mathcal{X}}{\jmath\omega\mu_\mathrm{i}} \right) \mathbf{e} \triangleq \mathcal{Y}\mathbf{e} \qquad \longrightarrow \qquad \overline{\overline{G}}\mathbf{J} = \left(\frac{\overline{\overline{X'}}}{\jmath\omega\mu_\mathrm{e}} - \frac{\overline{\overline{X}}}{\jmath\omega\mu_\mathrm{i}} \right) \mathbf{E} \triangleq \overline{\overline{Y}}\mathbf{E}$$

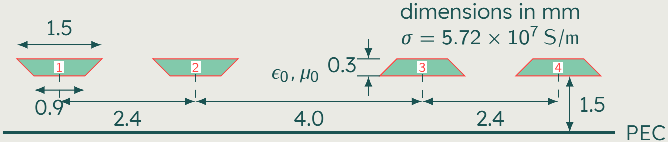
- Extract resistance and inductance matrices $\overline{\overline{R}}$ and $\overline{\overline{L}}$

$$\overline{\overline{R}} + \jmath\omega\overline{\overline{L}} = \left(\overline{\overline{T}}^\mathrm{T} \left(\overline{\overline{G}}\,\overline{\overline{Y}}^{-1}\overline{\overline{G}} + \jmath\omega\overline{\overline{A}} \right)^{-1} \overline{\overline{T}} \right)^{-1}$$

with $\overline{\overline{G}}$ a Gramian matrix, $\overline{\overline{A}}$ the interaction matrix and $\overline{\overline{T}}$ an incidence matrix

Application

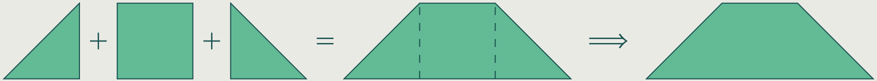
Asymmetric configuration with four trapezoidal conductors



T. Demeester and D. De Zutter, "Construction of the Dirichlet to Neumann boundary operator for triangles and applications in the analysis of polygonal conductors," IEEE Trans. Microw. Theory Techn., 2010

Improvements w.r.t. the reference

- No need to model trapezoid as a composition of a rectangle and two triangles
- No tweaking required to eliminate Gibbs effect in triangles



Application

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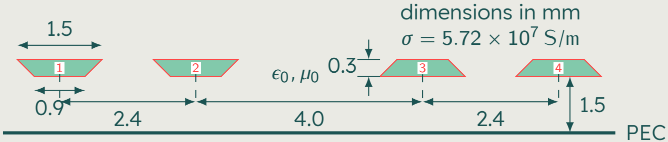

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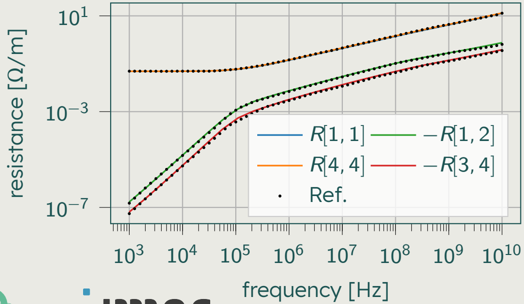
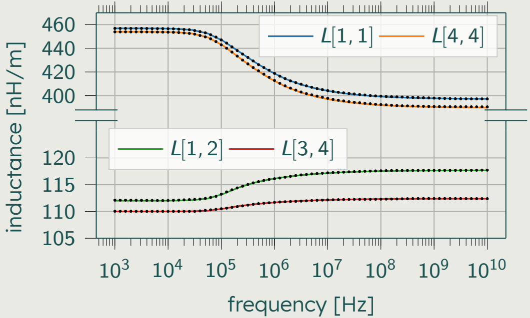
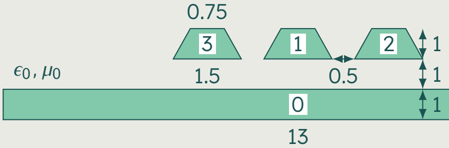




Application

Multiconductor transmission line with finite reference plane



dimensions in µm
$\sigma = 3.57 \times 10^7$ S/m

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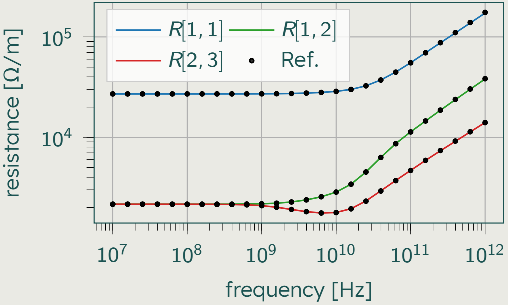
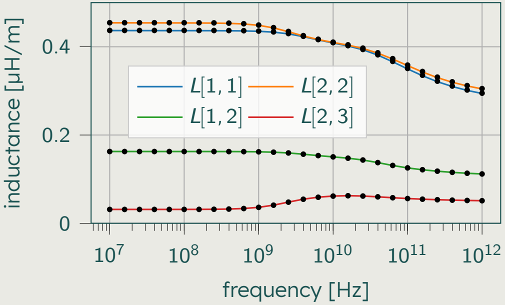
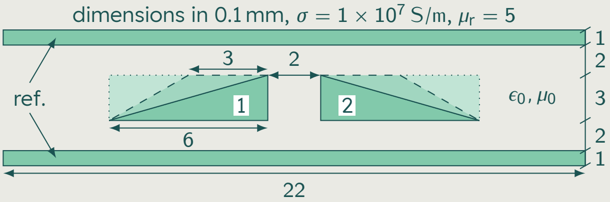




Application

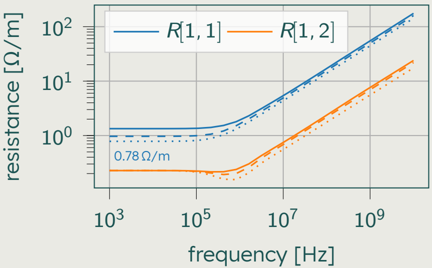
Influence of the conductor shape

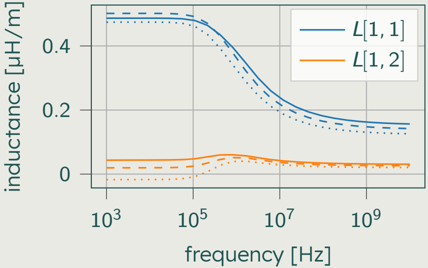


Pouillet resistances:

$$R_{11}^{DC} = \frac{1}{\sigma}\left(\frac{1}{A_{sig}} + \frac{1}{A_{ref}}\right)$$

$$R_{12}^{DC} = \frac{1}{\sigma A_{ref}}$$





Conclusion

Novel interconnect modeling framework

- Robust construction of differential surface admittance operator for **arbitrary polygonal cross-sections**
- Extension of **Fokas method** to complex eigenvalues to solve pertinent partial differential equations
- Efficient and accurate extraction of **per-unit-of-length resistance and inductance** parameters

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