Quantum
Mechanical &
Electromagnetic
Systems
Modelling Lab

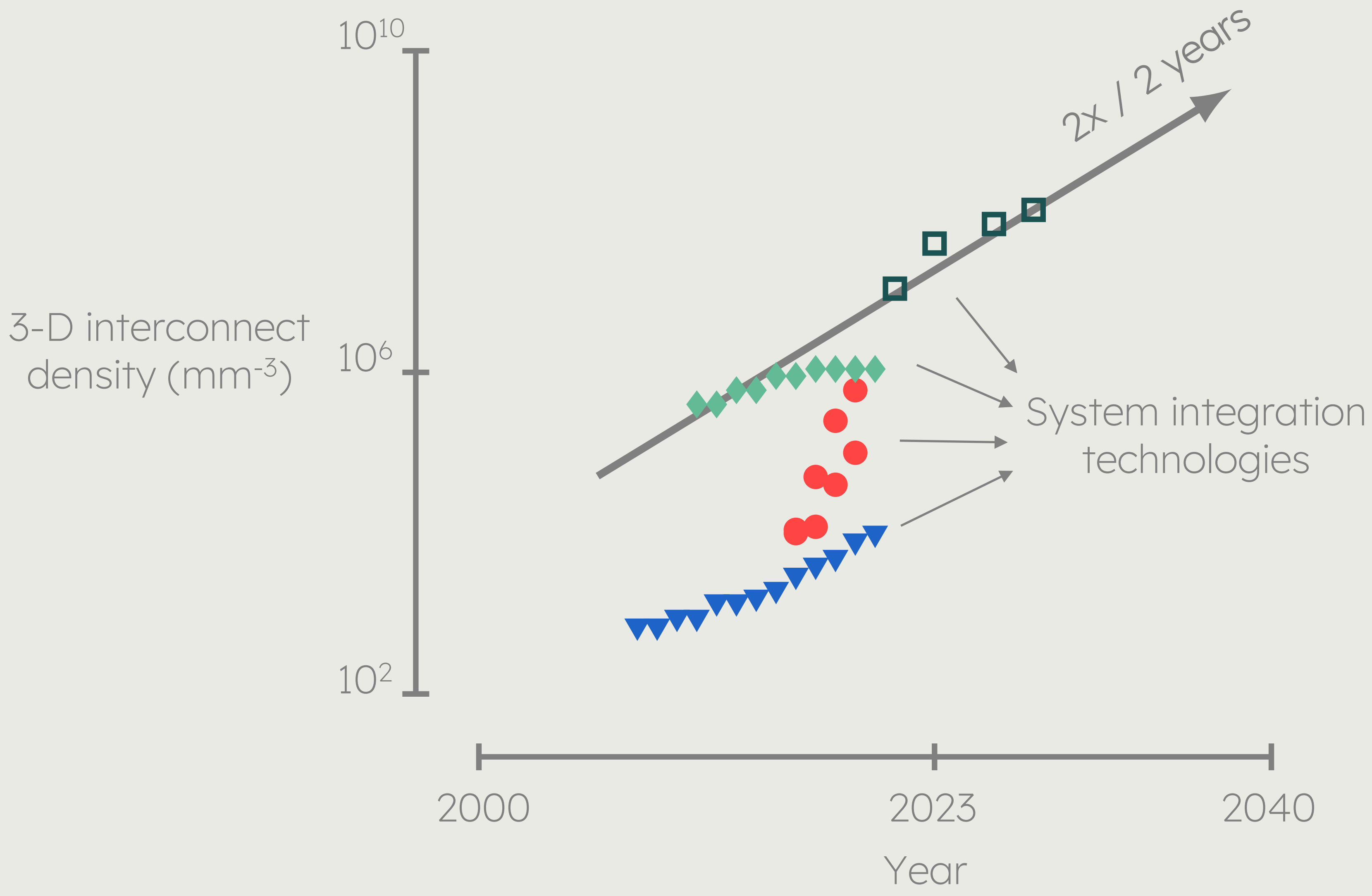
Fokas Based Dirichlet-to-Neumann Operators for Accurate Signal Integrity Assessment of Interconnects

Martijn Huynen, Dries Bosman, Daniël De Zutter, Dries Vande Ginste

quest.

NEMO 2023

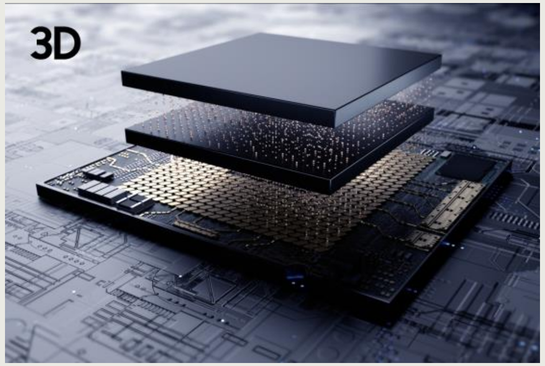
Interconnect scaling projected to keep doubling to accommodate bandwidth demand



Source: TSMC, ISCC 2021

New approaches and materials are required to meet this demand

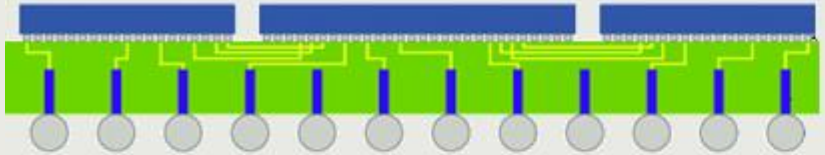
3-D Integrated circuits (3DICs) exploit the vertical dimension



Interposers and chiplets enable dense heterogenous integration

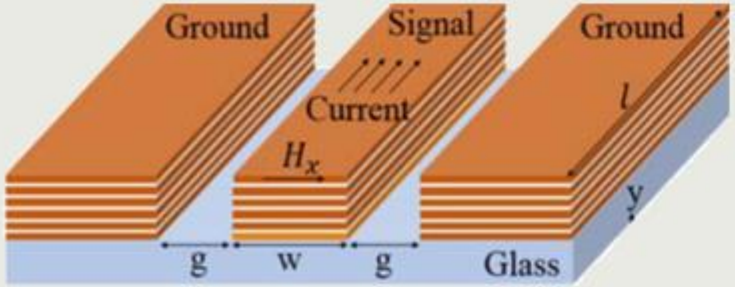
Chiplet

Interposer



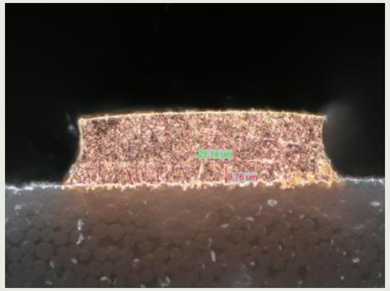
Application of more exotic materials, e.g., magnetic components, present opportunities and challenges

Metaconductor



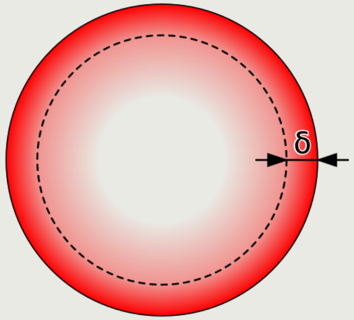
Advances in computational methods are needed to enable design methodologies

Miniaturization & increased operating frequency ask for more accurate and complete modeling



(Un)wanted intricate geometric details become increasingly impactful

Full wave solvers indispensable to capture effects such as the skin effect or the proximity effect



Challenging materials such as semiconductors or magnetic materials often not accurately modeled over large frequency ranges in (commercial) solvers

At quest, we developed a new approach to tackle these challenges

Full RLGC characterization of interconnects with arbitrary polygonal cross-sections

Broadband capturing of skin and proximity effects in conductors and magnetic materials

Applicable to emerging interconnect topologies

Assess signal integrity performance of novel solutions

The Dirichlet-to-Neumann operator

Fokas computation of the Laplace equation

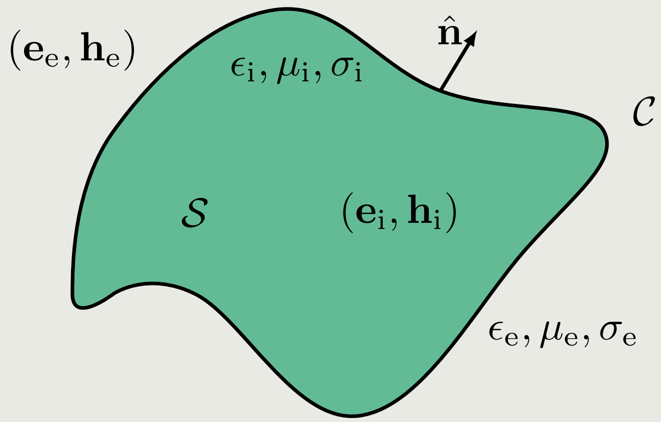
Interconnect analysis

Metaconductor performance

Future work

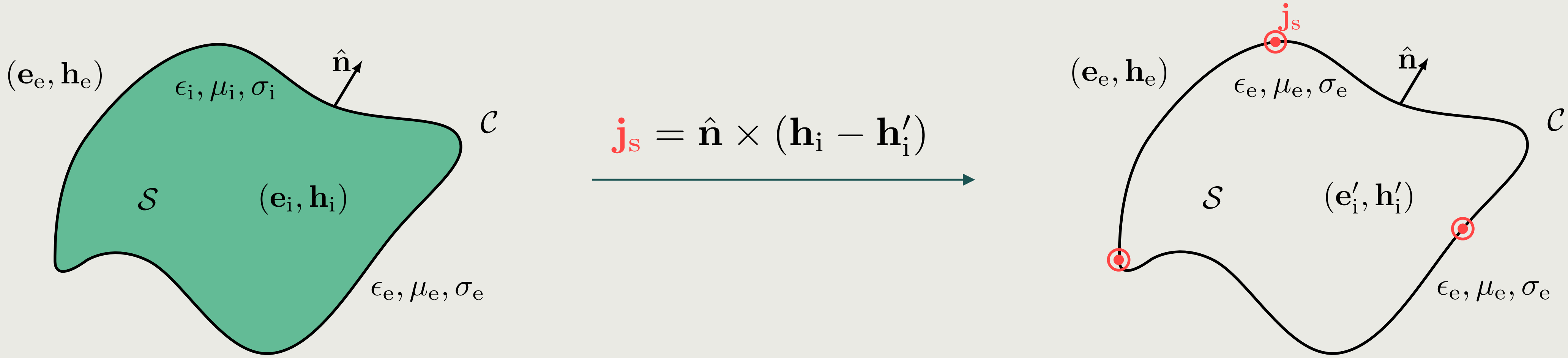
The equivalence principle replaces materials by the background medium

RL parameters



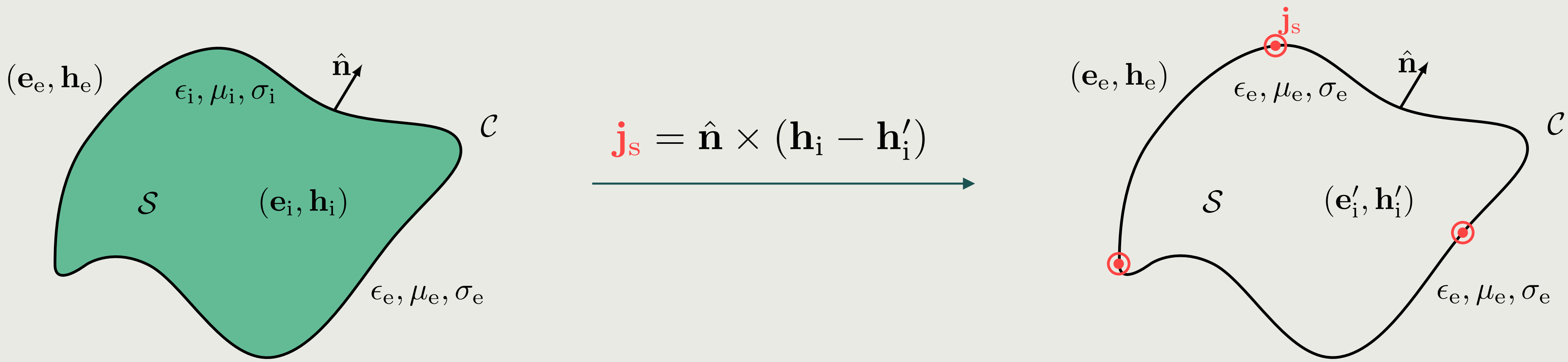
The equivalence principle replaces materials by the background medium

RL parameters

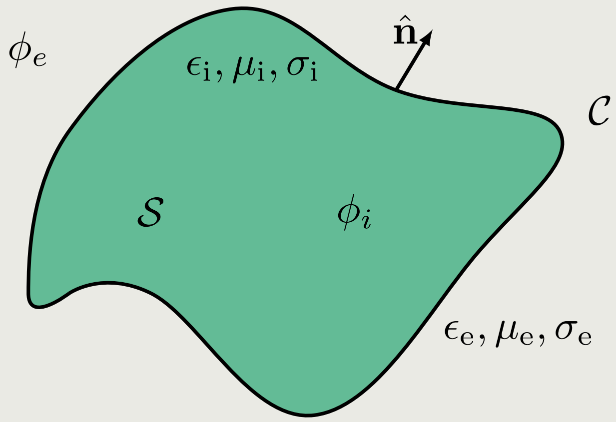


The equivalence principle replaces materials by the background medium

RL parameters

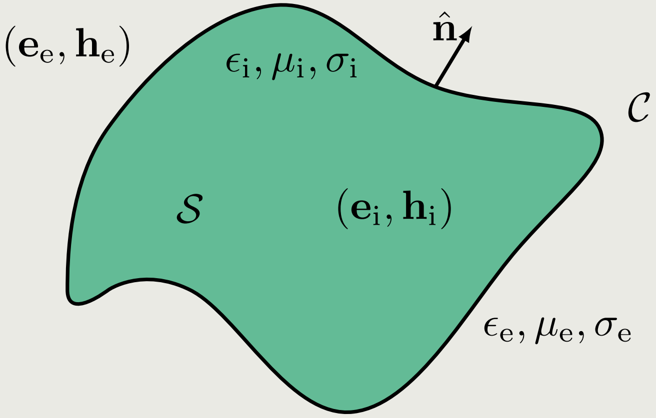


GC parameters

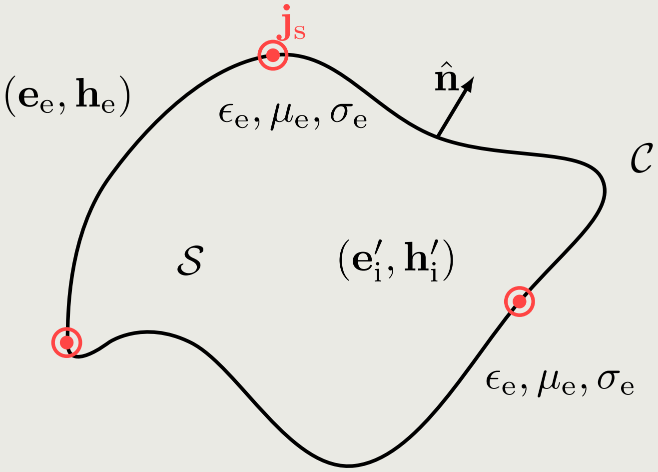


The equivalence principle replaces materials by the background medium

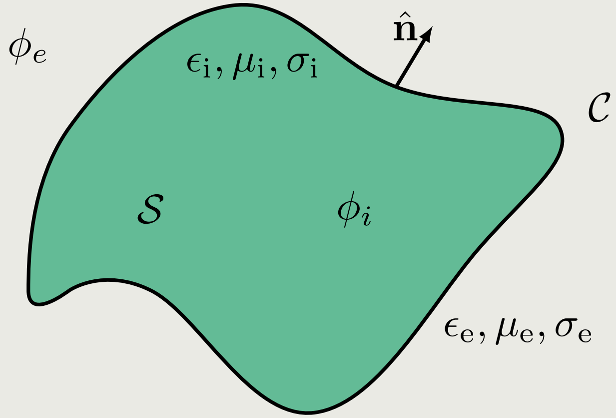
RL parameters



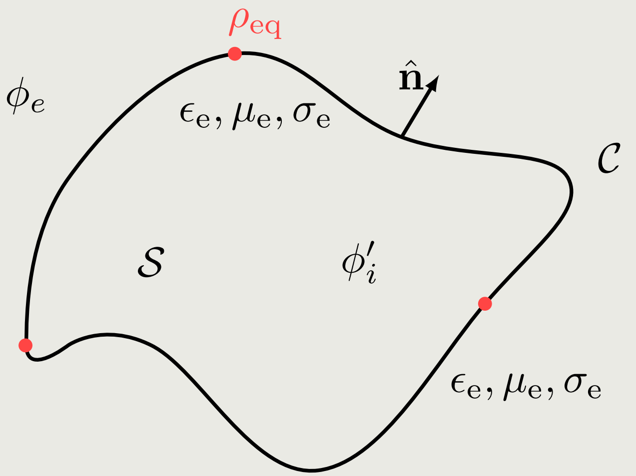
$$\mathbf{j}_\mathrm{s} = \hat{\mathbf{n}} \times (\mathbf{h}_\mathrm{i} - \mathbf{h}'_\mathrm{i})$$



GC parameters

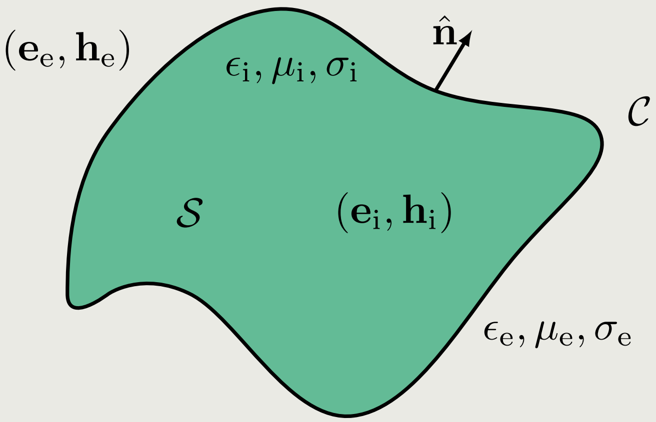


$$\rho_\mathrm{eq} = -(\epsilon_i - \epsilon_e) e_n$$

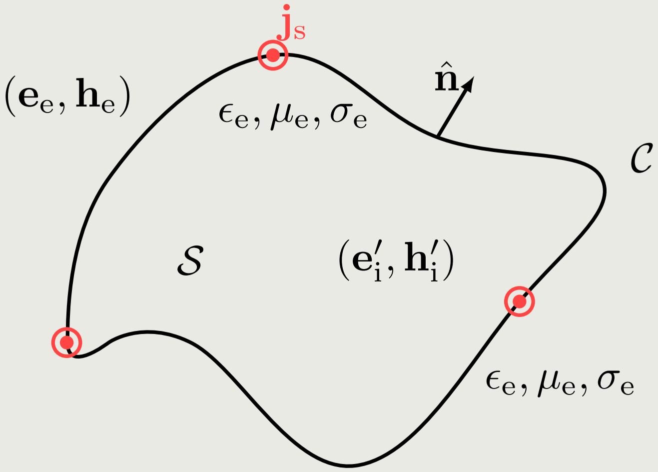


The equivalence principle replaces materials by the background medium

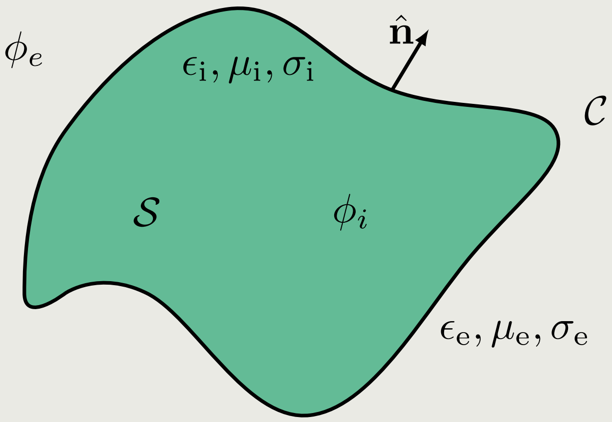
RL parameters



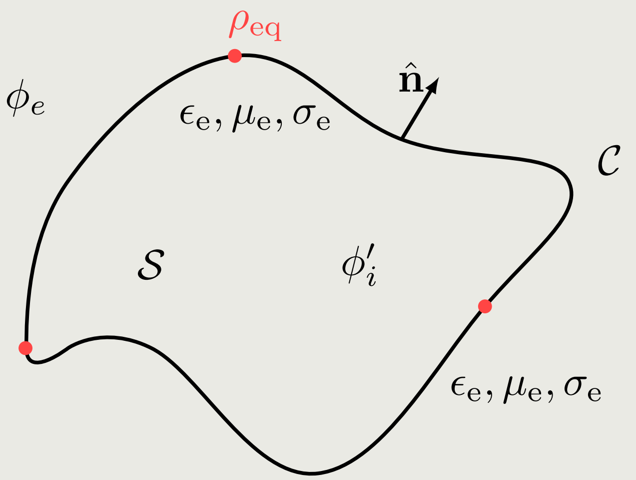
$$\mathbf{j}_\mathrm{s} = \hat{\mathbf{n}} \times (\mathbf{h}_\mathrm{i} - \mathbf{h}'_\mathrm{i})$$



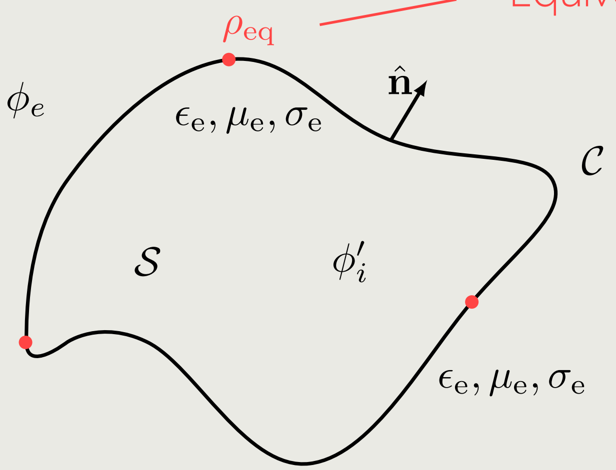
GC parameters



$$\rho_\mathrm{eq} = -(\epsilon_i - \epsilon_e) e_n$$

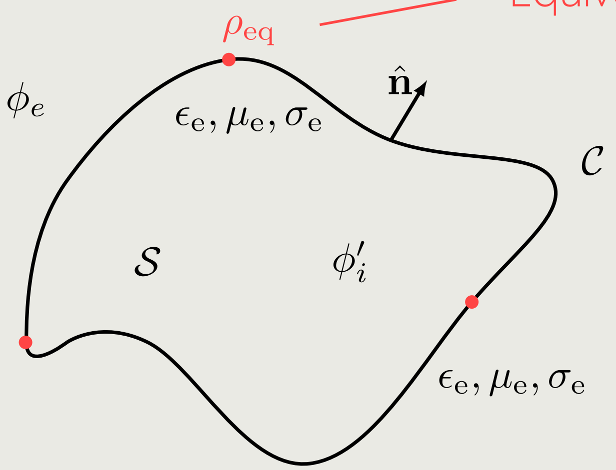


The capacitance matrix can be computed in the quasi-TM regime



Equivalent surface charge density $\rho_{\mathrm{eq}} = -(\epsilon_i - \epsilon_e)e_n$

The capacitance matrix can be computed in the quasi-TM regime

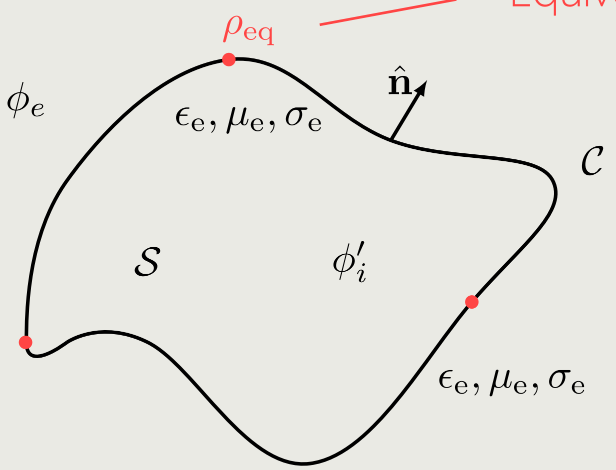


Equivalent surface charge density $\rho_{\mathrm{eq}} = -(\epsilon_i - \epsilon_e)e_n$

Quasi-TM

$$e_n = -\partial_n \phi$$

The capacitance matrix can be computed in the quasi-TM regime



Equivalent surface charge density $\rho_{\mathrm{eq}} = -(\epsilon_i - \epsilon_e)e_n$

Quasi-TM →

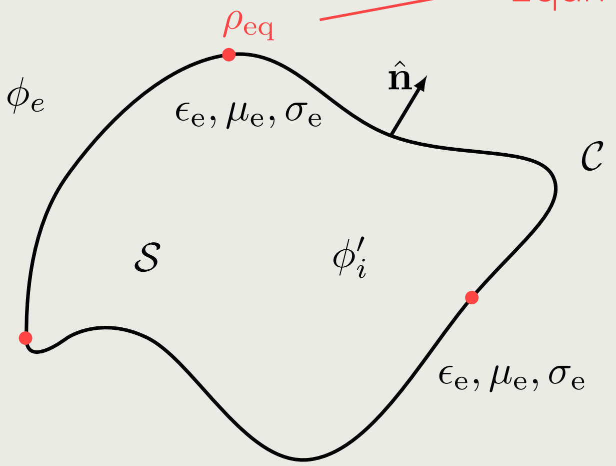
$$e_n = -\partial_n \phi$$

$$\partial_n \phi = \mathcal{D}\phi$$

Dirichlet-to-Neumann operator

Potential

The capacitance matrix can be computed in the quasi-TM regime



Equivalent surface charge density $\rho_{\mathrm{eq}} = -(\epsilon_i - \epsilon_e)e_n$

Quasi-TM

$$e_n = -\partial_n \phi$$

$$\partial_n \phi = \mathcal{D}\phi$$

Dirichlet-to-Neumann operator

Potential

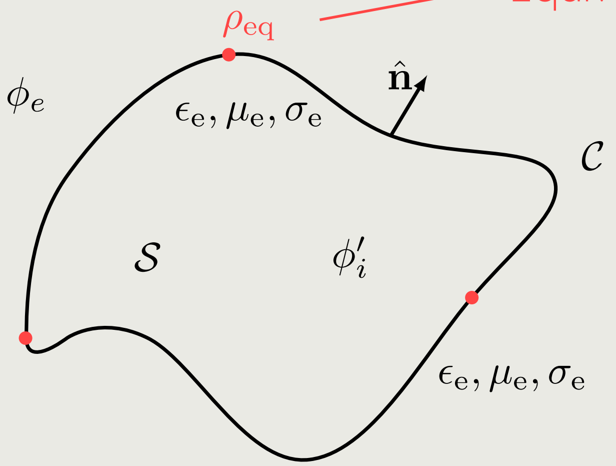
$$\rho_{\mathrm{eq}} = (\epsilon_i - \epsilon_e)\mathcal{D}\phi$$

$$\phi = -\frac{1}{\epsilon_e} \oint_{\mathcal{C}} \rho_{\mathrm{eq}}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}) \, \mathrm{d}\mathbf{r}'$$

Green's function

The capacitance matrix can be computed in the quasi-TM regime

Equivalent surface charge density $\rho_{\mathrm{eq}} = -(\epsilon_i - \epsilon_e)e_n$



Quasi-TM →

$$e_n = -\partial_n \phi$$

$$\partial_n \phi = \mathcal{D}\phi$$

Dirichlet-to-Neumann operator

Potential

$$\rho_{\mathrm{eq}} = (\epsilon_i - \epsilon_e)\mathcal{D}\phi$$

$$\phi = -\frac{1}{\epsilon_e} \oint_{\mathcal{C}} \rho_{\mathrm{eq}}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}) \, \mathrm{d}\mathbf{r}'$$

Green's function

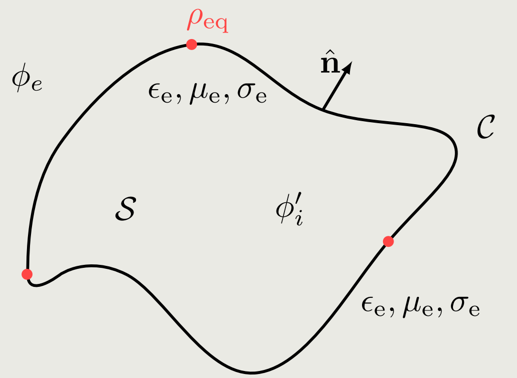
pulse basis functions

Galerkin discretization

$$\overline{\overline{C}} + \overline{\overline{G}}/j\omega$$

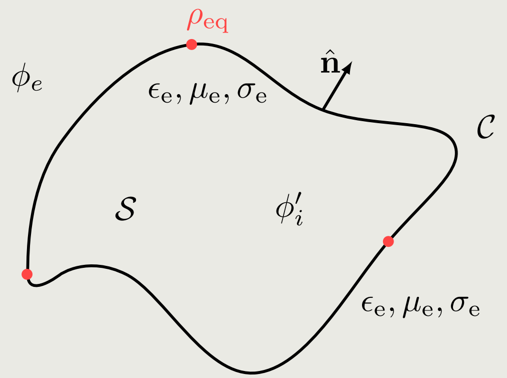
Capacitance matrix

The DtN-operator is calculated with the Fokas method or unified transform method



Inside $\mathcal{S}$, the potential ϕ obeys the Laplace equation $\nabla^2\phi = 0$

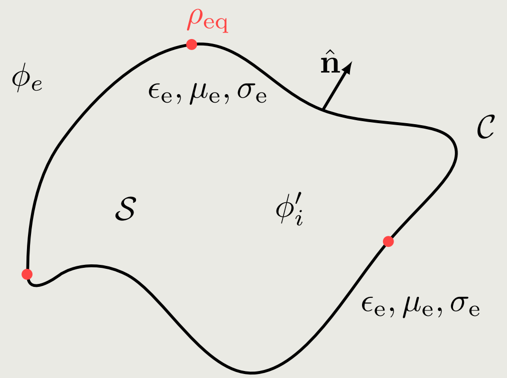
The DtN-operator is calculated with the Fokas method or unified transform method



Inside $\mathcal{S}$, the potential ϕ obeys the Laplace equation $\nabla^2\phi = 0$

On $\mathcal{C}$, the potential ϕ is "known" and we are looking for $\partial_n \phi$

The DtN-operator is calculated with the Fokas method or unified transform method



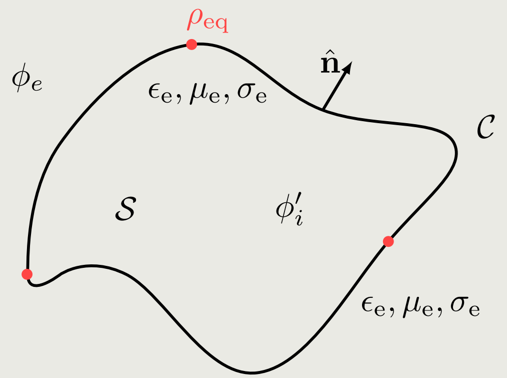
Inside $\mathcal{S}$, the potential ϕ obeys the Laplace equation $\nabla^2\phi = 0$

On $\mathcal{C}$, the potential ϕ is "known" and we are looking for $\partial_n \phi$

According to the **Fokas method**, this boundary value problem can be cast as a **global relation**

$$\oint_{\mathcal{C}} e^{-j\lambda\zeta}\left(\lambda\phi\,\mathrm{d}\zeta + \frac{\partial\phi}{\partial n}\,\mathrm{d}c\right) = 0 \qquad \zeta = x + jy$$

The DtN-operator is calculated with the Fokas method or unified transform method



Inside $\mathcal{S}$, the potential ϕ obeys the Laplace equation $\nabla^2\phi = 0$

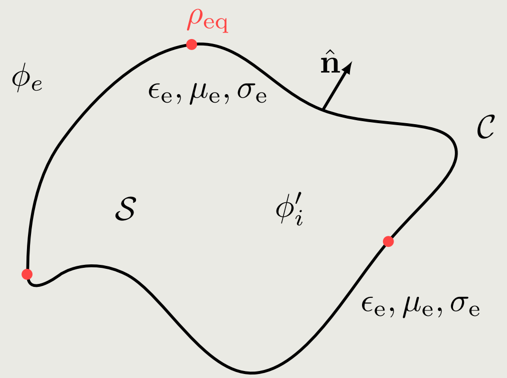
On $\mathcal{C}$, the potential ϕ is “known” and we are looking for $\partial_n \phi$

According to the **Fokas method**, this boundary value problem can be cast as a **global relation**

$$\oint_{\mathcal{C}} e^{-j\lambda\zeta} \left(\lambda\phi \, \mathrm{d}\zeta + \frac{\partial\phi}{\partial n} \, \mathrm{d}c\right) = 0 \qquad \zeta = x + jy$$

Dirichlet-to-Neumann (DtN)

The DtN-operator is calculated with the Fokas method or unified transform method



Inside $\mathcal{S}$, the potential ϕ obeys the Laplace equation $\nabla^2\phi = 0$

On $\mathcal{C}$, the potential ϕ is "known" and we are looking for $\partial_n\phi$

According to the **Fokas method**, this boundary value problem can be cast as a **global relation**

$$\oint_{\mathcal{C}} e^{-j\lambda\zeta}\left(\lambda\phi\,\mathrm{d}\zeta + \frac{\partial\phi}{\partial n}\,\mathrm{d}c\right) = 0 \qquad \zeta = x + jy$$

Dirichlet-to-Neumann (DtN)

Discretization of this relation by expanding ϕ and $\partial_n\phi$ into Legendre polynomials and selecting well-chosen spectral collocation points λ, leads to a discretized $\mathcal{D}$

The Dirichlet-to-Neumann operator

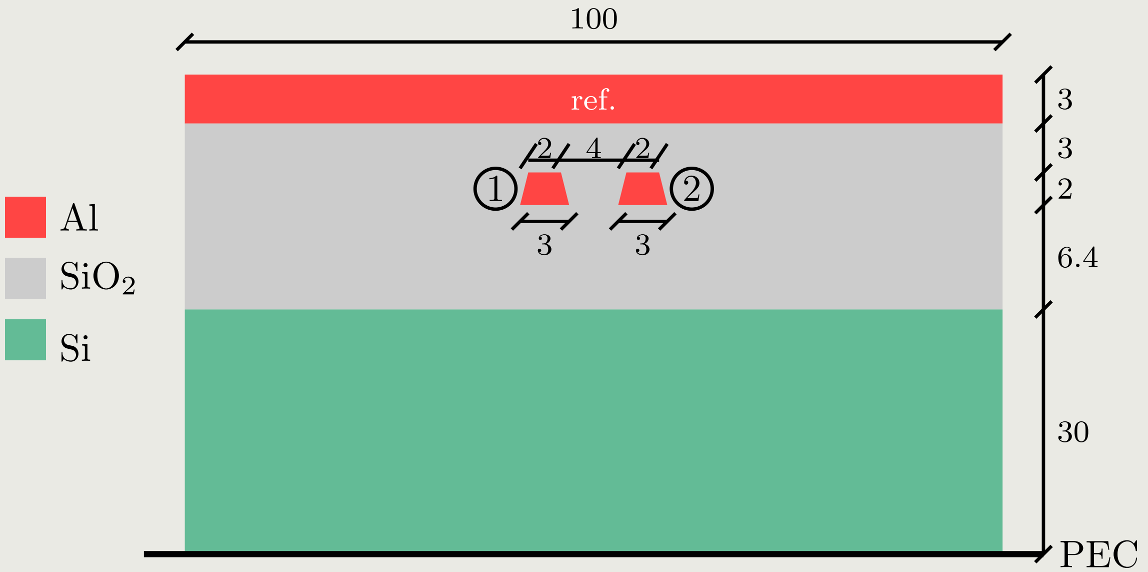
Fokas computation of the Laplace equation

Interconnect analysis

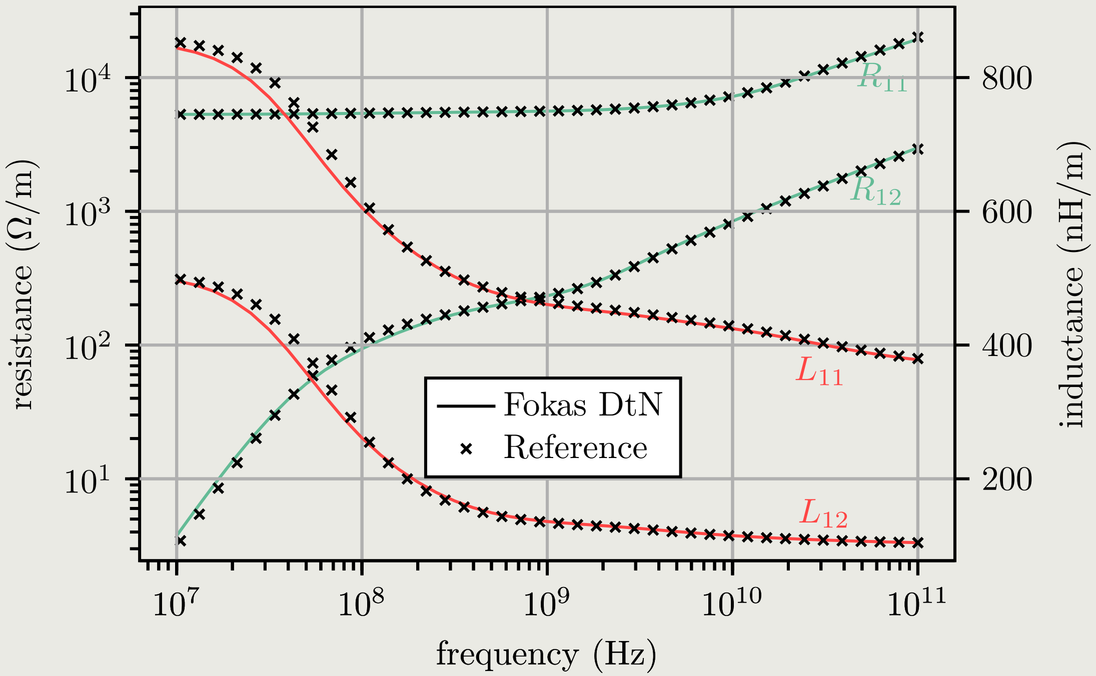
Metaconductor performance

Future work

A pair of coupled embedded lines shows excellent agreement with the reference result

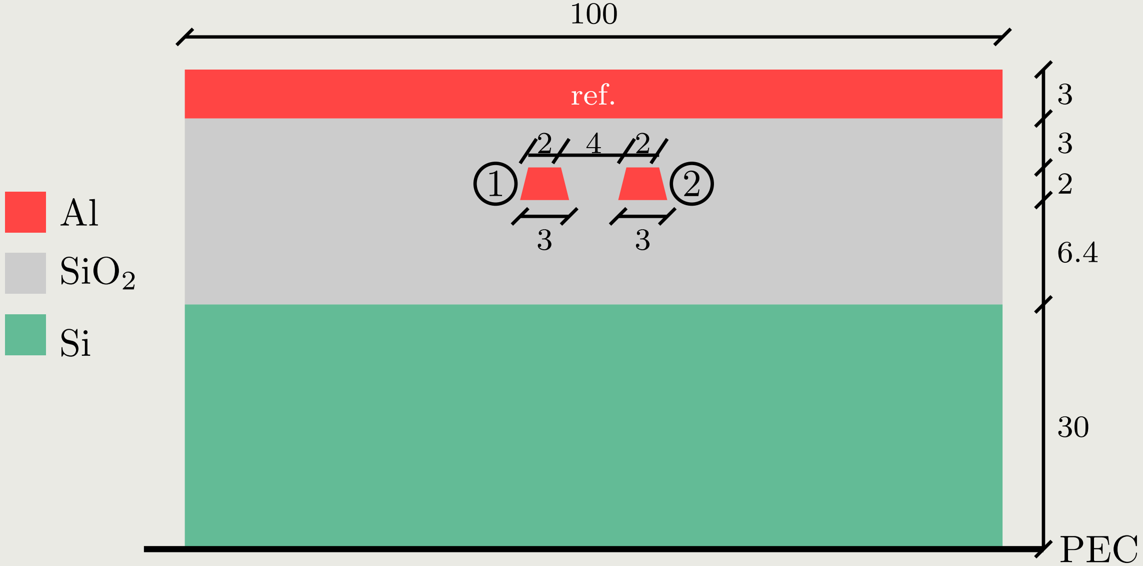


Dimensions in um

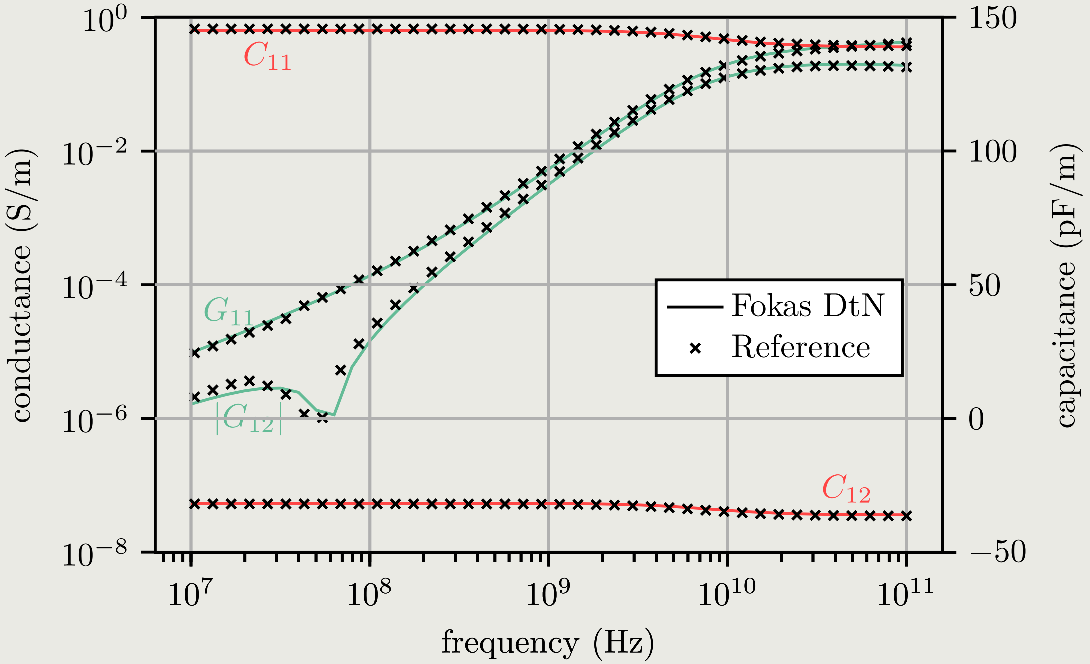


Reference:
D. Vande Ginste et al.:
URSI GASS 2011

A pair of coupled embedded lines shows excellent agreement with the reference result

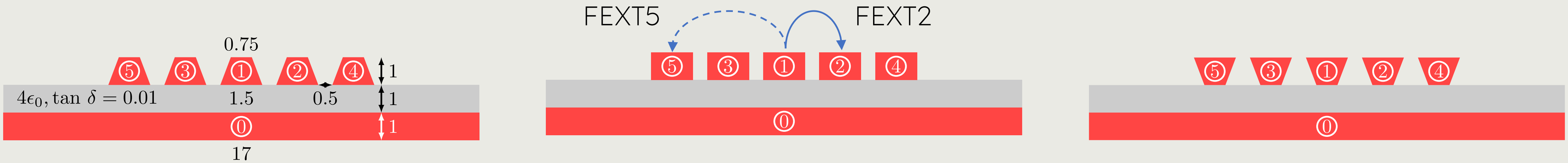


Dimensions in um

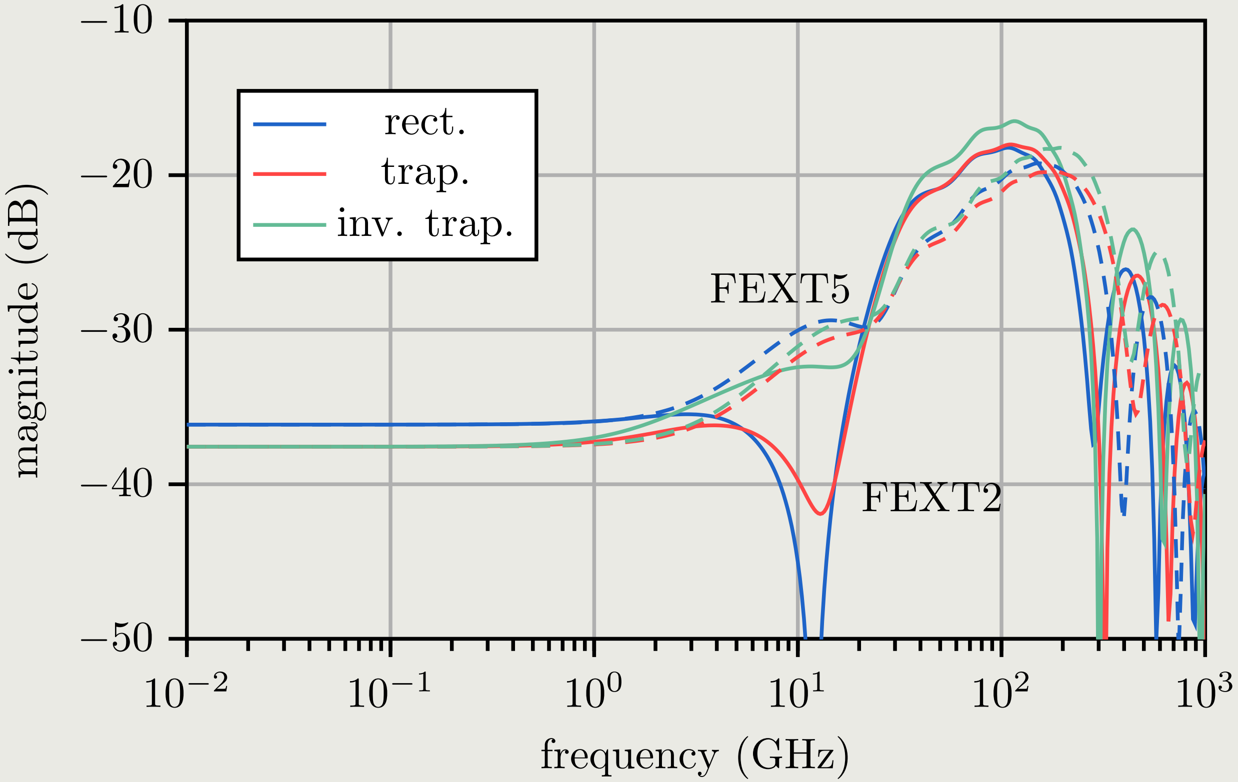


Reference:
D. Vande Ginste et al.:
URSI GASS 2011

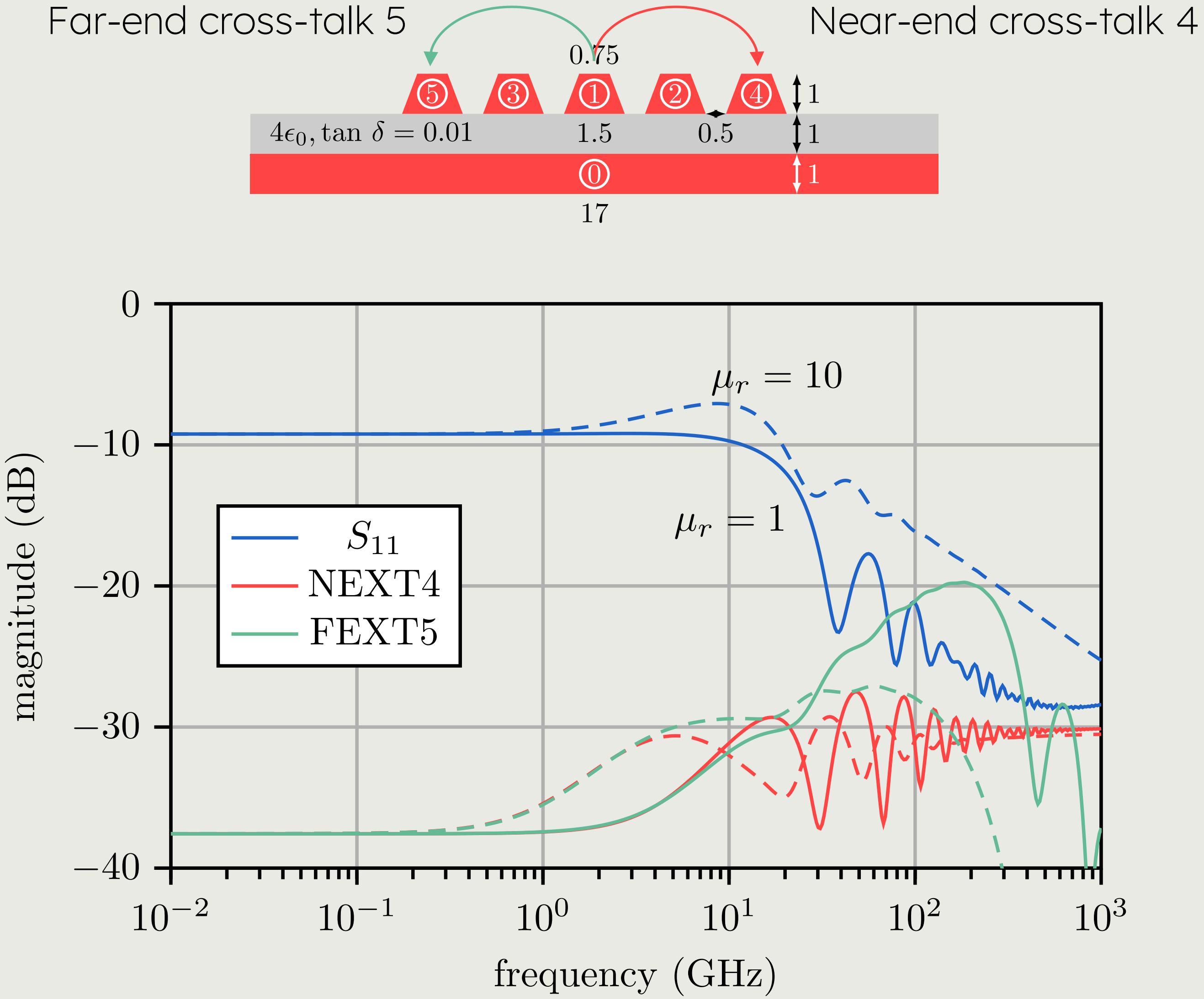
Far-end cross-talk (FEXT) in multiconductor TL strongly depends on conductor's shape



Dimensions in um



Magnetic materials heavily impact the signal integrity performance



The Dirichlet-to-Neumann operator

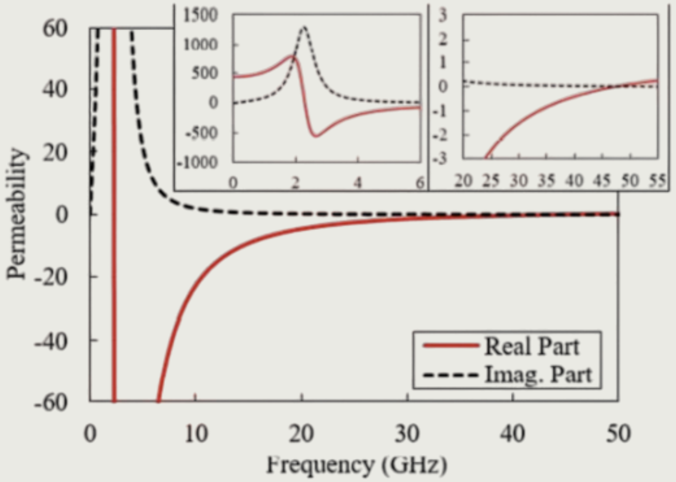
Fokas computation of the Laplace equation

Interconnect analysis

Metaconductor performance

Future work

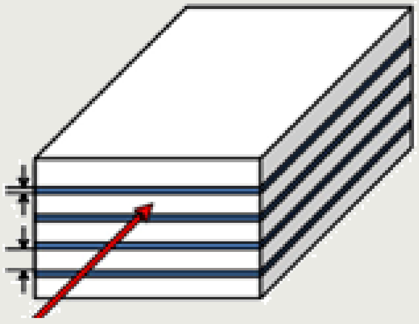
Alternating magnetic and non-magnetic layered conductors can reduce skin effect considerably



Materials such as cobalt exhibit a negative permeability in certain frequency ranges

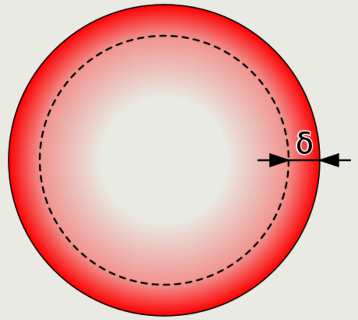
Alternating layers of non-magnetic conductors and magnetic material will exhibit an lower average $\mu_r < 1$ or even $\mu_r = 0$

$$\mu_{r,\text{av}} = \frac{1 \cdot t_{\text{cu}} + \mu_{r,\text{mag}} \cdot t_{\text{mag}}}{t_{\text{cu}} + t_{\text{mag}}}$$

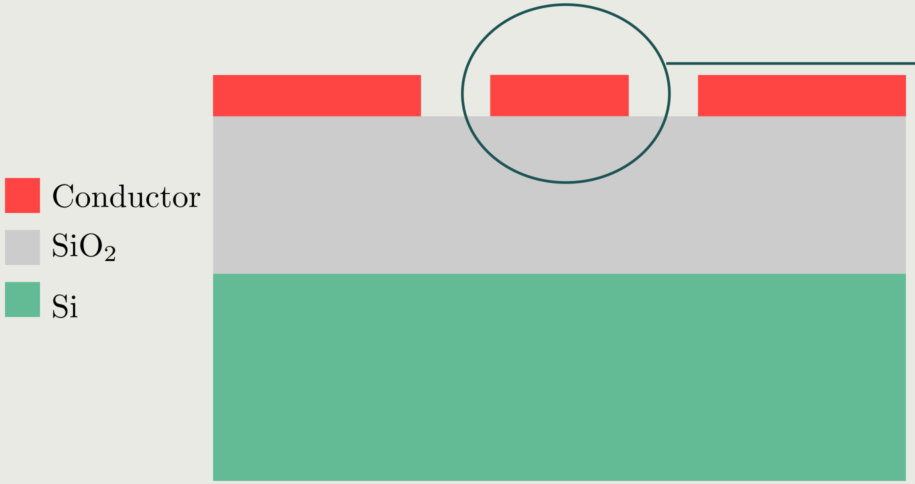


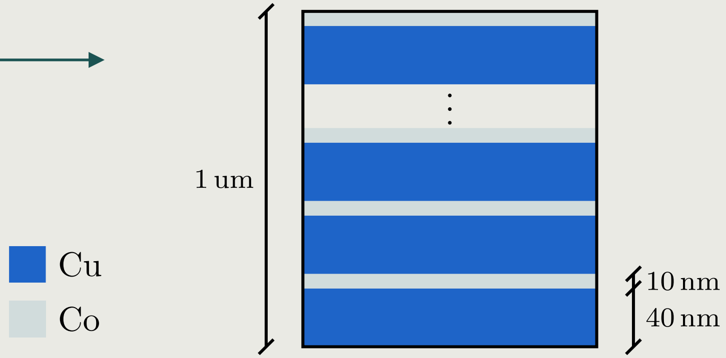
This reduces the skin effect and hence losses in interconnects

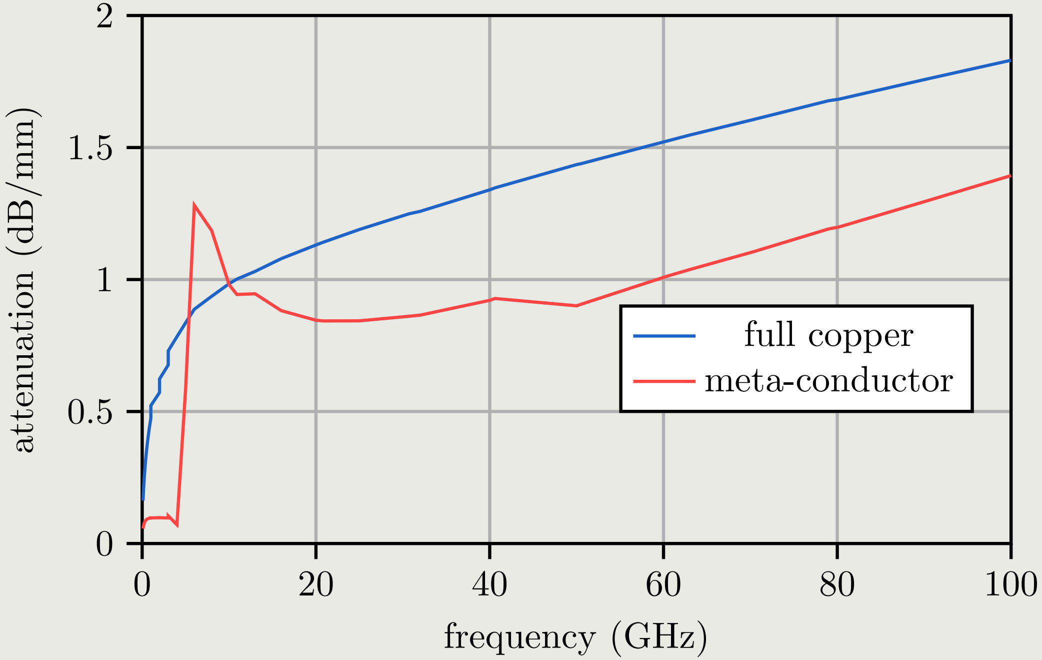
$$\delta = \sqrt{\frac{2}{\omega \sigma \mu_0 \mu_r}}$$



The reduced skin effect significantly lowers the attenuation over a large frequency range







The Dirichlet-to-Neumann operator

Fokas computation of the Laplace equation

Interconnect analysis

Metaconductor performance

Future work

Our work is never over

Non-convex polygonal cross-sections

Non-polygonal cross-sections

Acceleration of the matrix calculations

Extension of the Fokas method to 3-D

...

quest.

Quantum Mechanical &
Electromagnetic Systems
Modelling Lab

GHENT UNIVERSITY

imec

Technologiepark – Zwijnaarde 126, B-9052 Gent, Belgium
Martijn.huynen@ugent.be
www.QuestLab.be

Martijn HUYNEN, Post-doctoral researcher